UNIT V

Bonds

15m Explain Bonds and Notation

8.1 INTRODUCTION AND TERMINOLOGY

A *bond* is a written contract between the issuer (borrower) and the investor (lender), which specifies:

(i) The *face value*, or *denomination*, of the bond, which is stated on the front of the bond. This is usually a round figure such as \$100, \$500, \$1000, \$10 000.

(ii) The *redemption date*, or *maturity date*, which is the date on which the loan will be repaid.

(iii) The *bond rate*, or *coupon rate*, which is the rate at which the bond pays interest on its face value at regular time intervals until the maturity date. This rate is usually compounded semiannually.

(iv) The *redemption value*, which is the amount of money promised to be paid on the redemption date. In most cases, it is the same as the face value, and we say that the bond is *redeemed at par*.

Callable bonds contain a clause allowing the issuer to pay off the loan (redeem the bond) at a date earlier than the full redemption date. These are discussed in detail in Section 8.3.

Bonds may be bought and sold at any time. The buyer of a bond will want to realize a certain return on his investment, as specified by a desired *yield rate*.

In this chapter, we will use the following notation:

F ≡ face value or par value of the bond
C ≡ redemption value of the bond
r ≡ bond rate or coupon rate per interest period
i ≡ yield rate per interest period, often called the yield to maturity (YTM)
n ≡ number of interest periods until the redemption date
P ≡ purchase price of the bond to yield rate i
Fr ≡ bond interest payment or coupon

8.2 PURCHASE PRICE TO YIELD A GIVEN INVESTMENT RATE

The investor who wishes to realize a rate of return i (until the bond is redeemed or matures) should pay a price equal to the discounted value of the n coupons Fr plus the discounted value of the redemption amount C:

$$P = Fr\, a_{\overline{n}|i} + C(1+i)^{-n} \qquad (8.1)$$

In Problem 8.4, we show that (8.1) is equivalent to

$$P = C + (Fr - Ci)a_{\overline{n}|i} \tag{8.2}$$

known as an *alternate purchase-price formula*; computationally, (8.2) is a little simpler than (8.1).

SOLVED PROBLEMS

8.1 A \$1000 bond that pays interest at $j_2 = 12\%$ is redeemable at par at the end of 10 years. Find the purchase price to yield 10% compounded semiannually.

The bond pays $Fr = 1000(0.06) = \$60$ semiannually and \$1000 at the end of 10 years, as shown in Fig. 8-1.

$$P = 60a_{\overline{20}|.05} + 1000(1.05)^{-20} = 747.73 + 376.89 = \$1124.62$$

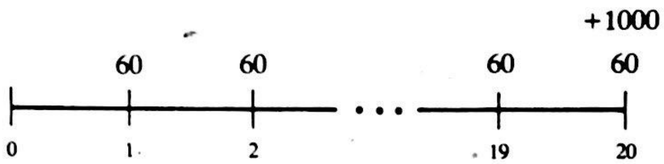

Fig. 8-1

8.2 Redo Problem 8.1 for a yield rate of $j_2 = 15\%$.

$$P = 60a_{\overline{20}|.075} + 1000(1.075)^{-20} = 611.67 + 235.41 = \$847.08$$

8.3 A \$5000 bond maturing at 103 on October 1, 2002, has semiannual coupons at $10\frac{1}{2}\%$. Find the purchase price on April 1, 1995, to yield $9\frac{1}{2}\%$ compounded semiannually.

The bond pays 15 semiannual coupons of $Fr = 5000(0.0525) = \$262.50$. The bond matures on October 1, 2002, for $C = 5000(1.03) = \$5150$. See Fig. 8-2.

$$P = 262.50a_{\overline{15}|.0475} + 5150(1.0475)^{-15} = 2771.29 + 2567.42 = \$5338.71$$

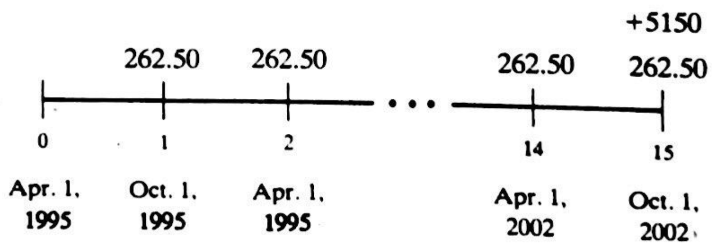

Fig. 8-2

8.4 Derive (8.2).

From (5.2),

$$a_{\overline{n}|i} = \frac{1-(1+i)^{-n}}{i} \qquad \text{or} \qquad (1+i)^{-n} = 1 - ia_{\overline{n}|i}$$

Replacing $(1+i)^{-n}$ in (8.1), we get:

$$P = Fr\, a_{\overline{n}|i} + C(1 - ia_{\overline{n}|i}) = C + (Fr - Ci)a_{\overline{n}|i}$$

8.5 Use (8.2) in (*a*) Problem 8.1, (*b*) Problem 8.2, (*c*) Problem 8.3.

(*a*) With $C = 1000$, $Fr = 60$, $i = 0.05$, and $n = 20$,

$$P = 1000 + (60 - 50)a_{\overline{20}|.05} = 1000 + 124.62 = \$1124.62$$

(*b*) With $C = 1000$, $Fr = 60$, $i = 0.075$, and $n = 20$,

$$P = 1000 + (60 - 75)a_{\overline{20}|.075} = 1000 - 152.92 = \$847.08$$

(*c*) With $C = 5150$, $Fr = 262.50$, $i = 0.0475$, and $n = 15$,

$$P = 5150 + (262.50 - 244.625)a_{\overline{15}|.0475} = 5150 + 188.71 = \$5338.71$$

8.6 A corporation issues a 15-year, \$10 000 par value bond with semiannual coupons at 10% per annum. Find the price to yield 9% per annum compounded monthly.

First, find a rate of interest, i, per half-year such that

$$\begin{aligned}(1+i)^2 &= \left(1+\frac{0.09}{12}\right)^{12} \\ 1+i &= (1.0075)^6 \\ i &= (1.0075)^6 - 1 = 0.045852235\end{aligned}$$

Then, by (8.2),

$$\begin{aligned}P &= 10\,000 + (500 - 458.52235)a_{\overline{30}|.045852235} \\ &= 10\,000 + 668.90 = \$10\,668.90\end{aligned}$$

8.7 Redo Problem 8.6 for a yield rate of $j_1 = 9\%$.

Find a rate of interest, i, per half-year such that

$$(1+i)^2 = 1.09 \qquad \text{or} \qquad i = 0.044030651$$

Then

$$\begin{aligned}P &= 10\,000 + (500 - 440.30651)a_{\overline{30}|.044030651} \\ &= 10\,000 + 983.53 = \$10\,983.53\end{aligned}$$

8.8 Derive *Makeham's purchase-price formula* for a bond redeemable at par,

$$P = F(1+i)^{-n} + \frac{r}{i}[F - F(1+i)^{-n}]$$

From (8.1), $P = Fr\, a_{\overline{n}|i} + F(1+i)^{-n}$. Substituting for $a_{\overline{n}|i}$,

$$P = Fr\frac{1-(1+i)^{-n}}{i} + F(1+i)^{-n} = F(1+i)^{-n} + \frac{r}{i}[F - F(1+i)^{-n}]$$

8.3 CALLABLE BONDS

Because *callable bonds* allow the issuer to pay off the loan (redeem the bond) prior to the maturity date, they present a problem with respect to the calculation of the purchase price, since the term of the bond is not certain. The investor will pay the price that will guarantee him his desired yield regardless of the call date. In determining the price, he must assume that the issuer of the bond will exercise his call option to the disadvantage of the investor.

For a bond *callable at par* ($C = F$):

If the yield rate is greater than the coupon rate, the investor must calculate using the **latest** possible call date. (See Problem 8.9.)

If the yield rate is less than the coupon rate, the investor must calculate using the **earliest** possible call date. (See Problem 8.10.)

For any callable bond, even if the bond is not callable at par ($C \neq F$), the investor can determine all possible purchase prices corresponding to his desired yield, and then pay the lowest of them. (See Problems 8.11 and 8.12.)

SOLVED PROBLEMS

8.9 The ABC Corporation issues a 20-year, $1000 bond with coupons at $j_2 = 12\%$. The bond can be called, at par, after 15 years. Find the purchase price to yield 13% compounded semiannually.

We must calculate the two purchase prices that correspond to the two possible redemption dates. If the bond is called after 15 years:

$$P_c = 1000 + (60 - 65)a_{\overline{30}|.065} = 1000 - 65.29 = \$934.71$$

If the bond matures after 20 years:

$$P_m = 1000 + (60 - 65)a_{\overline{40}|.065} = 1000 - 70.73 = \$929.27$$

The purchase price to guarantee a return of $j_2 = 13\%$ is the lower of these two answers, or $929.27. Paying that price, the investor realizes at least 13%, no matter what happens.

8.10 Redo Problem 8.9 for the yield rate $j_2 = 11\%$.

Again we calculate the two possible purchase prices. If the bond is called after 15 years:

$$P_c = 1000 + (60 - 55)a_{\overline{30}|.055} = 1000 + 72.67 = \$1072.67$$

If the bond matures after 20 years:

$$P_m = 1000 + (60 - 55)a_{\overline{40}|.055} = 1000 + 80.23 = \$1080.23$$

The price the investor must pay to guarantee a yield of $j_2 = 11\%$, regardless of what happens, is $1072.67.

8.11 For the bond of Problem 8.9, assume that the call feature specifies that if the bond is called after 15 years, the redemption value will be $1050. Otherwise, it matures at par in 20 years. Find the price to guarantee a yield of $j_2 = 11\%$.

If the bond is called after 15 years:

$$P_c = 1050 + (60 - 57.75)a_{\overline{30}|.055} = 1050 + 32.70 = \$1082.70$$

If the bond matures after 20 years, $P_m = \$1080.23$ (Problem 8.10). Hence, the price to guarantee a return of $j_2 = 11\%$ is $P_m = \$1080.23$.

8.12 A \$5000 callable bond pays interest at $j_2 = 9\frac{1}{2}\%$ and matures at par in 20 years. It may be called at the end of years 10 to 15 (inclusive) for \$5200. Find the price to yield at least $j_2 = 8\frac{1}{2}\%$ until the redemption.

If the bond is called after 10 years:

$$P_c = 5200 + (237.50 - 221)a_{\overline{20}|.0425} = \$5419.36$$

If the bond is called after 15 years:

$$P_c = 5200 + (237.50 - 221)a_{\overline{30}|.0425} = \$5476.85$$

Price P_c for call dates at the end of years 10 to 15 will gradually increase from \$5419.36 to \$5476.85.

If the bond matures after 20 years:

$$P_m = 5000 + (237.50 - 212.50)a_{\overline{40}|.0425} = \$5476.93$$

Hence, the price to guarantee a return of at least $j_2 = 8\frac{1}{2}\%$ until redemption is \$5419.36.

8.4 PREMIUM AND DISCOUNT

A bond is said to be purchased *at a premium* if its purchase price, P, exceeds its redemption value, C; the premium is $P - C$. A bond is said to be purchased *at a discount* if the purchase price, P, is less than its redemption value, C; the discount is $C - P$.

From (8.2), we can see that

$$\begin{aligned} \text{premium} &\equiv P - C = (Fr - Ci)a_{\overline{n}|i} \\ \text{discount} &\equiv C - P = (Ci - Fr)a_{\overline{n}|i} \end{aligned}$$

The *book value* of a bond at a given time is the sum recorded as being invested in the bond at that time. The book value of a bond on a date of purchase which coincides with an interest payment date (more precisely, the date of purchase precedes an interest payment date by one interest period) is just the purchase price of the bond. The book value on the redemption date is the redemption value of the bond.

When a bond is purchased at a premium ($P > C$), the book value of the bond will be written down (decreased) at each bond interest date, so that at the time of redemption the book value will equal the redemption value. This process is called *amortization of the premium* or *writing down*. When a bond is purchased at a discount ($C > P$), the book value of the bond will be written up (increased) at each bond interest date, so that at redemption the book value will equal the redemption value. This process is called *accumulation of the discount* or *writing up*.

A bond *amortization* (or *accumulation*) *schedule* shows the division of each bond coupon into its interest-yielded and principal-adjustment portions, together with the book value after each coupon is paid.

The payments made during the term of a bond can be regarded as loan payments made by the borrower (bond issuer) to the lender (the bondholder) to repay a loan amount equal to the purchase price of a bond. The bond purchase price is calculated as the discounted value of those payments (coupons plus redemption value) at a certain yield rate (the interest rate on the loan). Thus the bond transaction can be regarded as the amortization of a loan and an amortization schedule for the bond can be constructed like the general loan amortization schedule in Section 7.1. (See Problem 8.16.)

SOLVED PROBLEMS

8.13 A \$1000 bond, redeemable at par on December 1, 1998, pays semiannual coupons at $j_2 = 9\%$. The bond is bought on June 1, 1996. Find the purchase price and construct a bond schedule, if the desired yield is 8% compounded semiannually.

From (8.2), the purchase price on June 1, 1996, is

$$P = 1000 + (45 - 40)a_{\overline{5}|.04} = \$1022.26$$

Thus the bond is purchased at a premium of \$22.26. Table 8-1 gives the bond amortization schedule.

1022.26 × 8/100 =

Table 8-1

Date	Bond Interest Payment	Interest on Book Value at Yield Rate	Principal Adjustment	Book Value
June 1, 1996	0	0	0	1022.26
Dec. 1, 1996	45.00	40.89	4.11	1018.15
June 1, 1997	45.00	40.73	4.27	1013.88
Dec. 1, 1997	45.00	40.56	4.44	1009.44
June 1, 1998	45.00	40.38	4.62	1004.82
Dec. 1, 1998	45.00	40.19	4.81	1000.01
TOTALS	225.00	202.74	22.25	

8.14 Redo Problem 8.13 if the desired yield is $j_2 = 10\%$.

Now the purchase price on June 1, 1996, is

$$P = 1000 + (45 - 50)a_{\overline{5}|.05} = \$978.35$$

and so the bond is purchased at a discount of \$21.65. See Table 8-2.

Table 8-2

Date	Bond Interest Payment	Interest on Book Value at Yield Rate	Principal Adjustment	Book Value
June 1, 1996	0	0	0	978.35
Dec. 1, 1996	45.00	48.92	-3.92	982.27
June 1, 1997	45.00	49.11	-4.11	986.38
Dec. 1, 1997	45.00	49.32	-4.32	990.70
June 1, 1998	45.00	49.54	-4.54	995.24
Dec. 1, 1998	45.00	49.76	-4.76	1000.00
TOTALS	225.00	246.65	-21.65	

8.15 Verify the following properties of the bond schedules of Problems 8.13 and 8.14: (*a*) All book values can be reproduced using the purchase-price formula. (*b*) The total of the principal-adjustment column equals the original premium or discount. (*c*) Successive principal adjustments are in the ratio $1+i$, where i is the desired yield rate.

(*a*) Consider the book value on June 1, 1997, for the bond purchased to yield $j_2 = 8\%$ (Table 8-1). Using (8.1),

$$P = 45a_{\overline{3}|.04} + 1000(1.04)^{-3} = \$1013.88$$

or using (8.2),

$$P = 1000 + (45-40)a_{\overline{3}|.04} = \$1013.88$$

Consider the book value on December 1, 1997, for the bond purchased to yield $j_2 = 10\%$ (Table 8-2). From (8.1),

$$P = 45a_{\overline{2}|.05} + 1000(1.05)^{-2} = \$990.70$$

or from (8.2),

$$P = 1000 + (45-50)a_{\overline{2}|.05} = \$990.70$$

(*b*) In Problem 8.13, the premium is $22.26. The total of the principal-adjustment column of Table 8-1 is $22.25. The 1¢ error is due to roundoff. In Problem 8.14, the discount is $21.65 and the total of the principal-adjustment column of Table 8-2 is $21.65.

(*c*) In Table 8-1,

$$\frac{4.27}{4.11} \approx \frac{4.44}{4.27} \approx \frac{4.62}{4.44} \approx \frac{4.81}{4.62} \approx 1.04$$

In Table 8-2,

$$\frac{4.11}{3.92} \approx \frac{4.32}{4.11} \approx \frac{4.54}{4.32} \approx \frac{4.76}{4.54} \approx 1.05$$

If we carried full decimal accuracy, these relationships would be exact.

8.16 Construct the amortization schedule for the loan of (*a*) Problem 8.13; (*b*) Problem 8.14.

(*a*) Table 8-3 is the amortization schedule for the loan of Problem 8.13.

Table 8-3

Date	Payment	Interest at $i = 0.04$	Principal Repaid	Outstanding Principal
June 1, 1996	0	0	0	1022.26
Dec. 1, 1996	45.00	40.89	4.11	1018.15
June 1, 1997	45.00	40.73	4.27	1013.88
Dec. 1, 1997	45.00	40.56	4.44	1009.44
June 1, 1998	45.00	40.38	4.62	1004.82
Dec. 1, 1998	1045.00	40.19	1004.81	0.01*
TOTALS	1225.00	202.74	1022.25	

*The 1¢ error is due to roundoff.

(*b*) Table 8-4 is the amortization schedule for the loan of Problem 8.14.

Table 8-4

Date	Payment	Interest at $i = 0.05$	Principal Repaid	Outstanding Principal
June 1, 1996	0	0	0	978.35
Dec. 1, 1996	45.00	48.92	−3.92	982.27
June 1, 1997	45.00	49.11	−4.11	986.38
Dec. 1, 1997	45.00	49.32	−4.32	990.70
June 1, 1998	45.00	49.54	−4.54	995.24
Dec. 1, 1998	1045.00	49.76	995.24	0
TOTALS	1225.00	246.65	978.35	

8.17 A 20-year bond with annual coupons is bought at a premium to yield $j_1 = 9.5\%$. If the amount of amortization of the premium in the 3rd bond interest payment is \$50, determine the amount of amortization of the premium in the 14th payment.

The entries in the principal-adjustment column are in the ratio $1 + i = 1.095$. Thus, the amount of amortization of the premium in the 14th payment is

$$50(1.095)^{11} = \$135.68$$

8.18 A \$1000 bond, redeemable at 105 on October 1, 1997, pays semiannual coupons at $10\frac{1}{2}\%$. The bond is bought on April 1, 1995, to yield $j_{365} = 14\%$. Find the purchase price and construct a bond schedule.

Find i per half-year such that

$$(1+i)^2 = \left(1 + \frac{0.14}{365}\right)^{365} \quad \text{or} \quad i = 0.072493786$$

From (8.1), the purchase price on April 1, 1995, is

$$P = 52.50a_{\overline{5}|.072493786} + 1050(1.072493786)^{-5} = \$953.80$$

Table 8-5 is the bond schedule.

Table 8-5

Date	Bond Interest Payment	Interest on Book Value at Yield Rate	Principal Adjustment	Book Value
Apr. 1, 1995	0	0	0	953.80
Oct. 1, 1995	52.50	69.14	-16.64	970.44
Apr. 1, 1996	52.50	70.35	-17.85	988.29
Oct. 1, 1996	52.50	71.64	-19.14	1007.43
Apr. 1, 1997	52.50	73.03	-20.53	1027.96
Oct. 1, 1997	52.50	74.52	-22.02	1049.98
TOTALS	262.50	358.68	-96.18	

Briefly explain about

8.5 PRICE OF A BOND BETWEEN BOND INTEREST DATES

Suppose that a bond is purchased between bond interest dates to yield the buyer interest at rate i. Letting

P_0 ≡ price of a bond on the preceding bond interest date (just after a coupon has been paid)
k ≡ fractional part of an interest period that has elapsed ($0 < k < 1$)
P ≡ purchase price of the bond on the actual purchase date, called the *flat price*

the purchase price of the bond, using the *theoretical method*, that is, compound interest for the fractional part of an interest period, is given by

$$P = P_0(1 + i)^k \tag{8.3}$$

However, in reality, the *practical method*, that is, simple interest for the fractional part of an interest period, is used, and the purchase price is given by

$$P = P_0(1 + ki) \tag{8.4}$$

The practical method yields slightly larger values (see Problem 8.19) and it will be used in this Outline, unless stated otherwise.

The purchase price P may be considered as composed of two parts: the *market price*, Q, which is always equal to the book value of the bond, plus the *accrued bond interest*, I, as of the date of purchase. Defining

P_1 ≡ price of the bond on the next bond interest date (just after a coupon has beer paid)

we have

$$P_1 = (1 + i)P_0 - Fr \tag{8.5}$$

We can obtain the market price by linear interpolation between P_0 and P_1

$$Q = P_0 + k(P_1 - P_0) \tag{8.6}$$

The accrued bond interest is given by

$$I = kFr \tag{8.7}$$

When P_0 and P_1 are eliminated among (8.4), (8.5), and (8.6), the result is

$$P = Q + I \tag{8.8}$$

as stated above. (See Problem 8.20.)

If the actual purchase price P were quoted, there would be a big discontinuity in price at each coupon date, when the accrued bond interest would abruptly change from Fr to zero (see Fig. 8-3). Therefore, market price Q is quoted; or, rather market price of a \$100 bond, called the *market quotation*, q, is given. It is rounded off to the nearest eighth, but published in its decimal equivalent form.

SOLVED PROBLEMS

8.19 A \$2000 bond, redeemable at par on October 1, 1998, pays bond interest at $j_2 = 10\%$. Find the purchase price on June 16, 1996, to yield $j_2 = 9\%$ using (*a*) the theoretical method; (*b*) the practical method.

The preceding bond interest date is April 1, 1996. The exact time elapsed from April 1, 1996, to June 16, 1996, is 76 days. The exact time elapsed from April 1, 1996 to October 1, 1996, is 183 days. Thus, $k = \frac{76}{183}$.
We have $Fr = 2000(.05) = \$100$, $Ci = 2000(.045) = \$90$ and from (8.2)

$$P_0 = 2000 + (100 - 90)a_{\overline{5}|.045} = \$2043.90$$

(*a*) From (8.3) $P = P_0(1+i)^k = 2043.90(1.045)^{\frac{76}{183}} = \$2081.61$

(*b*) From (8.4) $P = P_0(1+ki) = 2043.90[1 + (\frac{76}{183})(.045)] = \$2082.10$

8.20 Show that (8.8) is equivalent to (8.4).

Substituting (8.5), (8.6), and (8.7) into (8.8)

$$\begin{aligned} P &= Q + I \\ &= P_0 + k(P_1 - P_0) + kFr \\ &= P_0 + kP_1 - kP_0 + kFr \\ &= P_0 + k[(1+i)P_0 - Fr] - kP_0 + kFr \\ &= P_0 + kP_0 + kiP_0 - kFr - kP_0 + kFr \\ &= P_0 + kiP_0 \\ &= P_0(1+ki) \end{aligned}$$

8.21 A \$1000 bond, redeemable at par on October 1, 1998, is paying bond interest at rate $j_2 = 9\%$. Find the purchase price on August 7, 1996, to yield 10% per annum compounded semiannually and determine the market price, accrued bond interest, and market quotation on August 7, 1996.

$$\begin{aligned} P_0(\text{on Apr. 1}) &= 1000 + (45 - 50)a_{\overline{5}|.05} = \$978.35 \\ P(\text{on Aug. 7}) &= (978.35)\left[1 + \frac{128}{183}(0.05)\right] = \$1012.57 \\ P_1(\text{on Oct. 1}) &= (1.05)(978.35) - 45 = \$982.27 \\ Q(\text{on Aug. 7}) &= 978.35 + \frac{128}{183}(982.27 - 978.35) = \$981.09 \end{aligned}$$

Scaling down the bond to \$100 gives us the market quotation

$$q = \frac{981.09}{10} = 98.11 \approx 98\frac{1}{8}$$

Finally $$I(\text{on Aug. 7}) = \frac{128}{183} \times 45 = \$31.48$$

As a check: $$Q + I = 981.09 + 31.48 = \$1012.57 = P$$

The relationship between the purchase price, P, the market price, Q, and the accrued bond interest, I, is shown in Fig. 8-3.

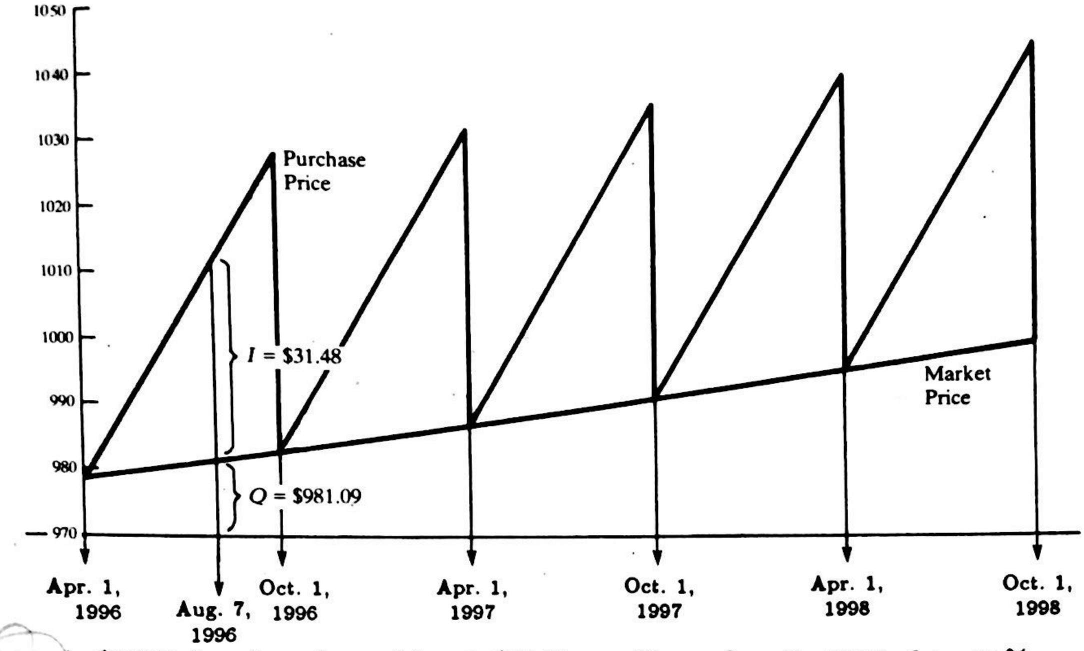

Fig. 8-3

8.22 A $1000 bond, redeemable at $1100 on November 9, 2006, has 11% coupons payable semiannually. Find the purchase price on April 29, 1996 if the desired yield is $j_\infty = 8\%$.

Find a rate of interest i per half-year such that

$$(1+i)^2 = e^{0.08}$$
$$i = e^{0.04} - 1 = 0.040810774$$

Then P_0 (Nov. 9, 1995) $= \$1100 + (55 - 1100i)a_{\overline{22}|i} = \$1244.95$. Now applying (8.4) and remembering that 1996 is a leap year,

$$P(\text{Apr. 29, 1996}) = 1244.95\left(1 + \frac{172}{182}i\right) = \$1292.97$$

8.23 A $10 000 bond with semiannual coupons at $9\frac{1}{2}\%$ is redeemable at par on August 25, 2005. This bond is sold on September 10, 1996, at a market quotation of $98\frac{7}{8}$. What did the buyer pay?

We arrange the data as in Fig. 8-4. The market price on September 10, 1996 is

$$Q = 100 \times 98\frac{7}{8} = \$9887.50$$

The accrued bond interest from August 25, 1996, to September 10, 1996, is

$$I = \frac{16}{184} \times 475 = \$41.30$$

Then, by (8.8), the total purchase price is

$$P = Q + I = 9887.50 + 41.30 = \$9928.80$$

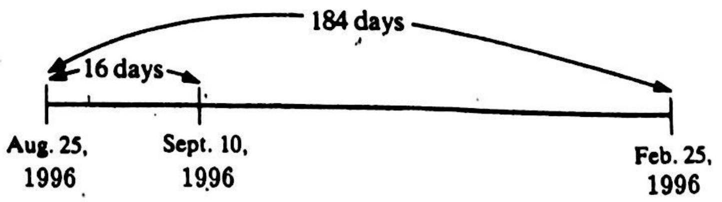

Fig. 8-4

8.24 A \$2000 bond, paying bond interest at $j_2 = 10\frac{1}{2}\%$, is redeemable at par on February 11, 2004. What should be the market quotation on November 25, 1995 to yield the buyer 9% per annum compounded semiannually?

We arrange the data as in Fig. 8-5. The purchase price on August 11, 1995, to yield $j_2 = 9\%$ is, by (8.2),

$$P_0 = 2000 + (105 - 90)a_{\overline{17}|.045} = \$2175.61$$

The purchase price on February 11, 1996, is, by (8.5),

$$P_1 = (1.045)(2175.61) - 105 = \$2168.51$$

The market price on November 25, 1995, is, by (8.6),

$$Q = 2175.61 + \frac{106}{184}(2168.51 - 2175.61) = \$2171.52$$

Reducing Q to a \$100 bond, we get the market quotation

$$q = \frac{2171.52}{20} = 108.58 \approx 108\frac{5}{8}$$

Alternate Solution

$$P(\text{Nov. } 25, 1995) = P_0(1 + ki) = 2175.61\left[1 + \frac{106}{184}(0.045)\right] = \$2232.01$$

The accrued bond interest from August 11, 1995 to November 25, 1995, is

$$I = \frac{106}{184} \times 105 = \$60.49$$

Then the market price on November 25, 1995, is

$$Q = P - I = 2232.01 - 60.49 = \$2171.52$$

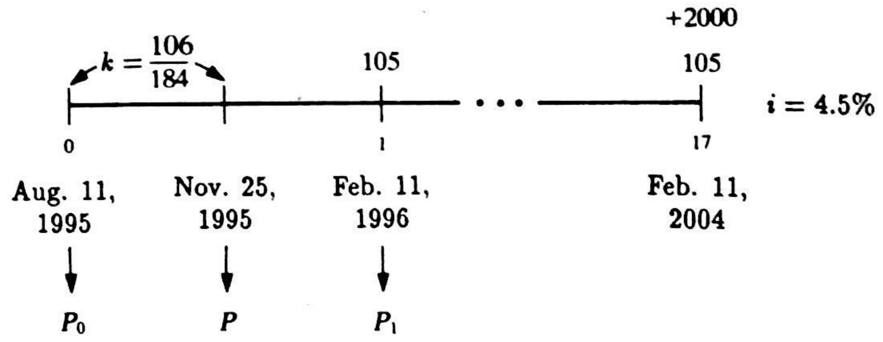

Fig. 8-5

8.6 FINDING THE YIELD RATE

Method of Averages (Bond Salesman's Method)

The yield rate per interest period (often called the yield to maturity) is approximated as

$$i \approx \frac{\text{average income per period}}{\text{average amount invested}} = \frac{(nFr + C - P)/n}{(P + C)/2}$$

(See Problem 8.24.)

If a more accurate answer is desired, the Method of Averages should be followed by the Method of Interpolation.

Method of Interpolation

This method requires determining market prices of a bond for two interest rates, such that one price is smaller and the other is greater than the given quoted price. Linear interpolation is then used to find i or j_m. If convenient, the interpolation can be on purchase price rather than market price.

The Method of Averages can be used to determine a starting point for the Method of Interpolation. (See Problem 8.26.)

SOLVED PROBLEMS

8.25 A \$2000 bond, paying semiannual coupons at $9\frac{1}{2}\%$ and redeemable at par on July 20, 2009, is quoted at $96\frac{1}{2}$ on July 20, 1995. Find an approximate value of the yield rate to maturity, j_2, using the Method of Averages.

The purchase price is $P = 20 \times 96\frac{1}{2} = \$1930$, since the bond is sold on a bond interest date. If held to maturity (28 periods), the buyer will have realized $28 \times 95 = \$2660$ in coupons, plus $2000 - 1930 = \$70$ in capital gains; or average income per period of

$$\frac{2660 + 70}{28} = \$97.50$$

The average amount invested is

$$\frac{1930 + 2000}{2} = \$1965$$

Thus, the approximate value of the yield rate per half-year is

$$i \approx \frac{97.50}{1965} = 0.0496 = 4.96\%$$

or $j_2 \approx 9.92\%$.

8.26 Taking the answer to Problem 8.25 as a starting point, find a more accurate answer using the Method of Interpolation.

We compute the market prices (which, here, are equal to the purchase prices) to yield $j_2 = 9\%$ and $j_2 = 10\%$.

$$Q(\text{to yield } j_2 = 9\%) = 2000 + (95 - 90)a_{\overline{28}|.045} = \$2078.71$$
$$Q(\text{to yield } j_2 = 10\%) = 2000 + (95 - 100)a_{\overline{28}|.05} = \$1925.51$$

Arranging the data in an interpolation table, we have:

		Q	j_2		
153.20	148.71	2078.71	9%	x	1%
		1930.00	j_2		
		1925.51	10%		

$$\frac{x}{1\%} = \frac{148.71}{153.20}$$

$$x = 0.97\%$$

$$j_2 \approx 9.97\%$$

Note $Q(\text{to yield } j_2 = 9.97\%) = 2000(95 - 99.70)a_{\overline{28}|.04985} = \$1929.86$, or $q = 96.493$.

8.27 A \$1000 bond with semiannual coupons at 12% matures at par on June 1, 2006. On February 3, 1996, this bond is quoted at $94\frac{7}{8}$. Find the approximate yield rate to maturity, using the Method of Averages.

Since we are looking only for an approximate answer, we may assume that the bond was quoted on the nearest coupon date, December 1, 1995. Then the investor will receive 21 coupons of \$60 each, plus capital gains of 1000 − 948.75 = \$51.25.

$$\begin{aligned}
\text{average income per period} &= \frac{21(60)+51.25}{21} = \$62.44 \\
\text{average amount invested} &= \frac{948.75+1000}{2} = \$974.38 \\
i &\approx \frac{62.44}{974.38} = 0.0641 = 6.41\% \\
j_2 &\approx 12.82\%
\end{aligned}$$

8.28 Find a more accurate estimate of the yield rate to maturity in Problem 8.27, by using the Method of Interpolation.

Select the two yield rates $j_2 = 12\%$ and $j_2 = 13\%$, and compute the corresponding market prices on February 3, 1996, using the time diagram given in Fig. 8-6.

At yield rate $j_2 = 12\%$, $Q = \$1000$ (since the yield rate coincides with the bond rate).

$$\begin{aligned}
\text{At } j_2 = 13\%, \quad P_0 &= 1000 + (60-65)a_{\overline{21}|.065} = \$943.58 \\
P_1 &= (1.065)(943.58) - 60 = \$944.91 \\
Q &= 943.58 + \frac{64}{182}(944.91 - 943.58) = \$944.05
\end{aligned}$$

Arranging the data in an interpolation table, we have:

	Q (on Feb. 3, 1996)	j_2	
55.95 { 51.25 {	1000.00	12%	} x } 1%
	948.75	j_2	
	944.05	13%	

$$\frac{x}{1\%} = \frac{51.25}{55.95}$$

$$x = 0.92\%$$

$$j_2 \approx 12.92\%$$

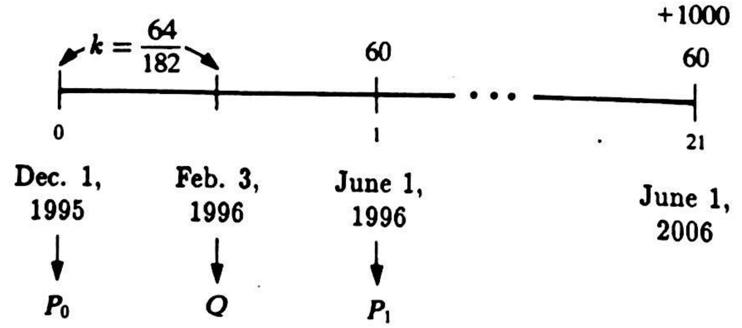

Fig. 8-6

8.29 A \$1000 bond, redeemable at par in 20 years, has semiannual coupons at 11%. It is callable at \$1050 at the end of 15 years. If it is quoted at 96, find the yield rate, j_2, using the Method of Interpolation and assuming (*a*) it is called, (*b*) it is not called.

(*a*) We want a yield rate $j_2 = 2i$ such that

$$960 = 1050 + (55 - 1050i)a_{\overline{30}|i}$$

By trial and error, we find

$$\text{at } j_2 = 11\%; \quad P = 1050 + (55 - 57.75)a_{\overline{30}|.055} = \$1010.03$$
$$\text{at } j_2 = 12\%; \quad P = 1050 + (55 - 63)a_{\overline{30}|.06} = \$939.88$$

Arranging the data in an interpolation table, we have:

		P	j_2		
70.15	50.03	1010.03	11%	x	1%
		960.00	j_2		
		939.88	12%		

$$\frac{x}{1\%} = \frac{50.03}{70.15}$$
$$x = 0.71\%$$
$$j_2 \approx 11.71\%$$

(*b*) We want a yield rate $j_2 = 2i$ such that

$$960 = 1000 + (55 - 1000i)a_{\overline{40}|i}$$

We know that at $j_2 = 11\%$ (which equals the bond rate), $P = \$1000$; and, at $j_2 = 12\%$,

$$P = 1000 + (55 - 60)a_{\overline{40}|.06} = \$924.77$$

Arranging the data in an interpolation table, we have:

		P	j_2		
75.23	40.00	1000.00	11%	x	1%
		960.00	j_2		
		924.77	12%		

$$\frac{x}{1\%} = \frac{40.00}{75.23}$$
$$x = 0.53\%$$
$$j_2 \approx 11.53\%$$

8.30 Mr. X buys a \$5000 bond that pays semiannual coupons at 10% and is redeemable at par in 10 years. The price he pays will provide a yield of $j_2 = 13\%$ if he holds the bond to maturity. After 5 years, Mr. X sells this bond to Mrs. Y, whose desired yield is $j_2 = 11\%$. (*a*) What price did Mr. X pay? (*b*) What price did Mrs. Y pay? (*c*) What yield, j_2, did Mr. X realize?

(*a*) $$P_X = 5000 + (250 - 325)a_{\overline{20}|.065} = \$4173.61$$

(*b*) $$P_Y = 5000 + (250 - 275)a_{\overline{10}|.055} = \$4811.56$$

(*c*) We want to find $j_2 = 2i$ such that

$$4173.61 = 250a_{\overline{10}|i} + 4811.56(1+i)^{-10}$$

By trial and error, we find

$$\text{at } j_2 = 14\%; \quad 250a_{\overline{10}|.07} + 4811.56(1.07)^{-10} = \$4201.85$$
$$\text{at } j_2 = 15\%; \quad 250a_{\overline{10}|.075} + 4811.56(1.075)^{-10} = \$4050.56$$

Arranging the data in an interpolation table, we have

$$151.29\left\{\begin{array}{l} 28.24\left\{\begin{array}{c|c} P & j_2 \\ \hline 4201.85 & 14\% \\ 4173.61 & j_2 \end{array}\right\}x \\ \begin{array}{c|c} 4050.56 & 15\% \end{array} \end{array}\right\}1\%$$

$$\frac{x}{1\%} = \frac{28.24}{151.29}$$

$$x = 0.19\%$$

$$j_2 \approx 14.19\%$$

8.7 OTHER TYPES OF BONDS

Serial Bonds

To borrow money, companies sometimes issue a series of bonds with staggered redemption dates instead of with a common redemption date. Such bonds are called *serial bonds*. Serial bonds can be thought of simply as several bonds covered under one bond indenture. If the redemption date of each individual bond is known, then the valuation of any one bond can be performed by methods already described. The value of the entire issue of the bonds is just the sum of the values of the individual bonds. (See Problem 8.31.)

Strip Bonds

Investors have several reasons for choosing one investment over another. They may look for cash flows whose timing matches their needs. For some investors, the coupons on a bond match their cash flow needs (for example, a corporation responsible for paying monthly retirement benefits to workers). For other investors, the redemption value of a bond matches their cash flow needs (for example, an insurance company which needs to satisfy the maturity value of an endowment insurance contract).

For these reasons, some investors separate the coupons from the redemption value of the bond. An investor may buy the bond contract, "strip" the coupons from the bond and sell the remainder of the asset, that is, the redemption value. This redemption-value-only bond is called a "strip" bond. (See Problem 8.34.) The original buyer may also sell each coupon as a separate asset. (See Problem 8.35.)

Annuity Bonds

An *annuity bond*, with face value F, is a contract promising the payment of an annuity whose present value is F at the bond rate. When the face value and the bond rate are given, the periodic payment R of the bond is computed by the methods developed in Chapters 5 and 6. At any date, the price of the annuity bond is obtained as the present value of the future payments of the bond at the investor's interest rate. (See Problems 8.36 and 8.37.)

SOLVED PROBLEMS

8.31 The directors of a company authorized the issuance of \$30 000 000 of serial bonds on September 1, 1994. Interest is to be paid annually on September 1 at rate 9%. The indenture provides that (i) \$5 000 000 of the issue is to be redeemed on September 1, 1999; (ii) \$10 000 000 of the issue is to be redeemed on September 1, 2004; (iii) \$15 000 000 of the issue is to be redeemed on September 1, 2009. Find the purchase price of the issue on September 1, 1994, that would yield $j_1 = 8\%$.

We consider the serial issue to be composed of 3 subissues, as shown in Fig. 8-7. The purchase price of the entire serial issue, P, on September 1, 1994, is the sum of the purchase prices of the 3 subissues, $P^{(1)}$, $P^{(2)}$, $P^{(3)}$.

$$\begin{aligned} P^{(1)} &= 5\,000\,000 + (450\,000 - 400\,000)a_{\overline{5}|.08} &&= 5\,199\,635.50 \\ P^{(2)} &= 10\,000\,000 + (900\,000 - 800\,000)a_{\overline{10}|.08} &&= 10\,671\,008.14 \\ P^{(3)} &= 15\,000\,000 + (1\,350\,000 - 1\,200\,000)a_{\overline{15}|.08} &&= \underline{16\,283\,921.70} \\ & & P &= \$32\,154\,565.34 \end{aligned}$$

8.32 What would be the value of the serial bonds of Problem 8.31 on September 1, 2001?

On the given date, the first subissue will have already been redeemed. Therefore:

$$\begin{aligned} P^{(2)} &= 10\,000\,000 + (900\,000 - 800\,000)a_{\overline{3}|.08} &&= 10\,257\,709.70 \\ P^{(3)} &= 15\,000\,000 + (1\,350\,000 - 1\,200\,000)a_{\overline{8}|.08} &&= \underline{15\,861\,995.83} \\ & & P &= \$26\,119\,705.53 \end{aligned}$$

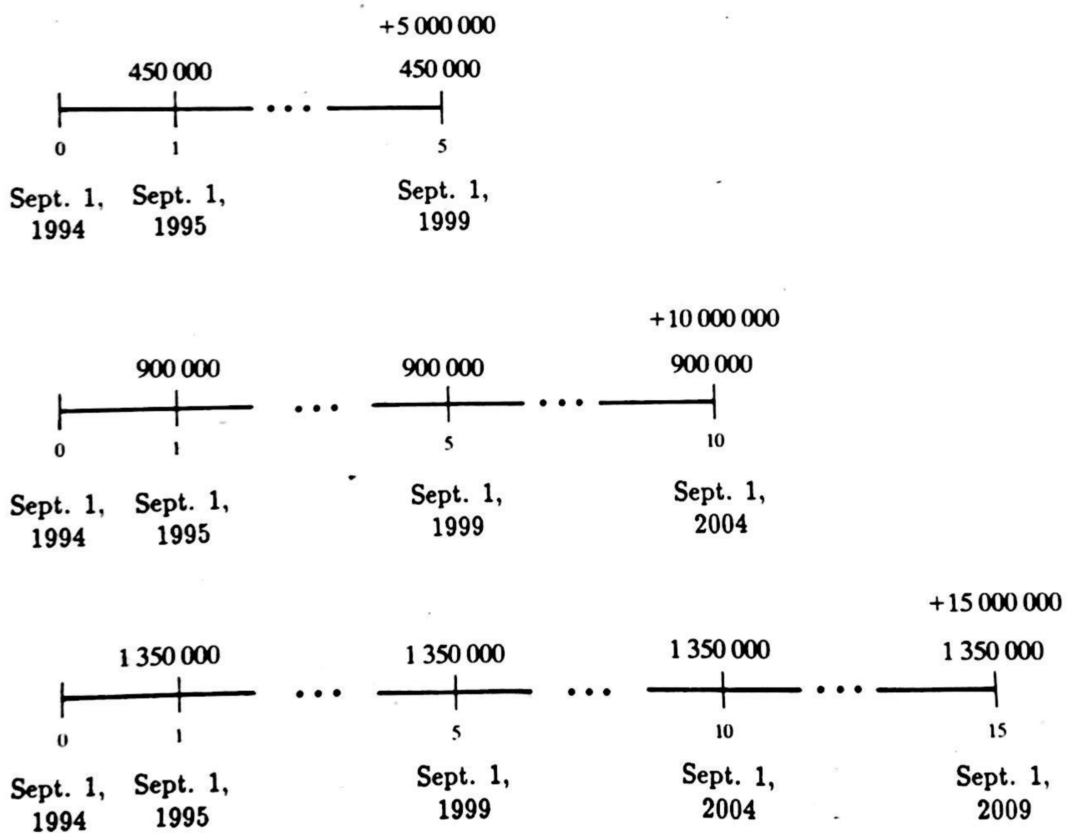

Fig. 8-7

8.33 The extension of Makeham's formula (Problem 8.8) to an entire serial issue, redeemable at par, is

$$P = \sum_k F^{(k)}(1+i)^{-t_k} + \frac{r}{i}\left[\sum_k F^{(k)} - \sum_k F^{(k)}(1+i)^{-t_k}\right]$$

where

$$P \equiv \text{purchase price of the entire issue}$$
$$F^{(k)}(1+i)^{-t_k} \equiv \text{present value of the } k\text{th redemption}$$
$$F^{(k)} \equiv \text{face value of the } k\text{th subissue}$$

Rework Problem 8.31 by means of this formula.

We have $r = 0.09$; $i = 0.08$; $F^{(1)} = 5\ 000\ 000$, $t_1 = 5$; $F^{(2)} = 10\ 000\ 000$, $t_2 = 10$; $F^{(3)} = 15\ 000\ 000$, $t_3 = 15$. Then, $\sum_k F^{(k)} = 30\ 000\ 000$,

$$\sum_k F^{(k)}(1+i)^{-t_k} = 5\ 000\ 000(1.08)^{-5} + 10\ 000\ 000(1.08)^{-10} + 15\ 000\ 000(1.08)^{-15} = \$12\ 763\ 476$$

and
$$P = 12\ 763\ 476 + \frac{0.09}{0.08}(30\ 000\ 000 - 12\ 763\ 476) = \$32\ 154\ 565$$

8.34 A \$1000 corporate bond, paying bond interest at $j_2 = 9\%$, is redeemable at par in 20 years. Investor A buys the bond to yield $j_2 = 8.5\%$. Investor A strips the coupons from the bond and sells the remaining "strip" bond to Investor B, who wishes a yield rate of $j_{12} = 9\%$. Find (*a*) the price Investor B paid for the strip bond; (*b*) the yield rate j_2 realized by Investor A.

(*a*) Investor B will pay the discounted value of \$1000 payable in 20 years at $j_{12} = 9\%$, or $i = 0.0075$ per month over 240 months.

$$P_B = 1000(1.0075)^{-240} = \$166.41$$

(*b*) First we calculate the price Investor A paid for the bond. With $F = C = 1000$, $r = 0.045$, $i = 0.0425$, and $n = 40$,

$$P_A = 1000 + (45 - 42.50)a_{\overline{40}|.0425} = \$1047.69$$

In return for the investment of \$1047.69, Investor A gets 40 coupons worth \$45 each over 20 years plus \$166.41 payable immediately from Investor B. We want to find a rate i per half-year, such that

$$1047.69 = 166.41 + 45a_{\overline{40}|i} \quad \text{or} \quad a_{\overline{40}|i} = 19.5840$$

Arranging the data in an interpolation table, we have:

		$a_{\overline{40}\|i}$	j_2		
1.3912	.2088	19.7928	8%	x	1%
		19.5840	j_2		
		18.4016	9%		

$$\frac{x}{1\%} = \frac{.2088}{1.3919}$$
$$x = .15\%$$
$$j_2 \approx 8.15\%$$

8.35 Find the price Investor C would pay in Problem 8.34 if Investor A sells the coupons to Investor C whose desired yield is $j_{12} = 7.5\%$.

Investor C will pay the discounted value of 40 semiannual coupons worth \$45 each, at $j_{12} = 7.5\%$, or $i = (1 + \frac{0.075}{12})^6 - 1$

$$P_C = 45a_{\overline{40}|i} = \$916.55$$

8.36 An annuity bond promises to repay \$50 000 principal with interest at $j_2 = 12\%$ in equal instalments at the end of each half-year for 10 years. How much will an investor offer for this bond if he wants to realize $j_2 = 13\%$?

First we find the semiannual payment, R, of an ordinary simple annuity:

$$R = \frac{50\ 000}{a_{\overline{20}|.06}} = \$4359.23$$

The price, P, the investor will pay is equal to the present value, at $j_2 = 13\%$, of the remaining payments (here, of all payments):

$$P = 4359.23a_{\overline{20}|.065} = \$48\ 032.21$$

8.37 For the annuity bond of Problem 8.36, find the purchase price with 5 years remaining on the bond, if the prospective investor's desired yield is $j_{365} = 10\%$.

Find rate i per half-year such that

$$(1+i)^2 = \left(1 + \frac{0.10}{365}\right)^{365} \quad \text{or} \quad i = 0.051263897$$

Then $$P = 4359.23a_{\overline{10}|i} = \$33\ 455.17$$

Supplementary Problems

PURCHASE PRICE TO YIELD A GIVEN INVESTMENT RATE

8.38 Find the purchase price of each bond in Table 8-6, using (8.1).

Table 8-6

	Face Value	Redemption	Bond Interest Rate, j_2	Years to Redemption	Yield Rate
(*a*)	\$5 000	at par	11%	20	$j_2 = 14\%$
(*b*)	1 000	at par	10%	15	$j_2 = 9\%$
(*c*)	2 000	at par	12%	10	$j_4 = 13\%$
(*d*)	500	at par	$10\frac{1}{2}\%$	20	$j_{12} = 12\%$
(*e*)	1 000	at 105	9%	18	$j_2 = 13\%$
(*f*)	5 000	at 110	12%	7	$j_\infty = 10\%$
(*g*)	10 000	at 103	$12\frac{1}{2}\%$	14	$j_{12} = 9\%$
(*h*)	2 000	at 110	10%	$2\frac{1}{2}$	$j_2 = 12\frac{1}{2}\%$

Ans. (*a*) \$4000.12; (*b*) \$1081.44; (*c*) \$1889.81; (*d*) \$443.58; (*e*) \$729.37; (*f*) \$5676.82; (*g*) \$12 681.51; (*h*) \$2043.10

8.39 Redo Problem 8.38, using (8.2).

8.40 The ACME Corporation needs to build a new plant. It issues \$500 000 worth of 20-year bonds with semiannual coupons at 11%. These bonds are redeemable at 105. The entire issue is purchased by an insurance company, whose required yield rate is $j_2 = 14\%$. Find the purchase price. *Ans.* \$401 681.69

8.41 Redo Problem 8.40, assuming now that the bonds are redeemable at par, using Makeham's purchase-price formula (Problem 8.8). *Ans.* \$400 012.18

8.42 (*a*) Mr. Green buys \$2000 bond paying bond interest at $j_2 = 10\frac{1}{2}\%$ and redeemable at par in 20 years to yield $j_4 = 12\%$. Find the purchase price. (*b*) After exactly 5 years, he sells the bond to another investor, whose desired yield rate is $j_1 = 9\frac{1}{2}\%$. Find the sale price.
Ans. (*a*) \$1750.06; (*b*) \$2194.72

8.43 The XYZ Corporation needs to raise some funds to pay for new equipment. They issue \$1 000 000 worth of 20-year bonds with semiannual coupons at $j_2 = 12\%$. These bonds are redeemable at 105. At the time of issue, interest rates in the market place are $j_{12} = 10.5\%$. How much money did they raise? *Ans.* \$1 109 694.04

8.44 A corporation issues \$600 000 worth of 12-year bonds with semiannual coupons at $j_2 = 10\%$. The bonds are priced to yield $j_2 = 9\%$. Determine the issue price per \$100 unit. *Ans.* \$107.25

8.45 A \$1000 bond paying coupons at $j_2 = 12\%$ is redeemable at par in 20 years. Find the price to yield an investor (*a*) $j_2 = 14\%$; (*b*) $j_2 = 12\%$; (*c*) $j_2 = 10\%$.
Ans. (*a*) \$866.68; (*b*) \$1000; (*c*) \$1171.59

8.46 An \$$X$ bond quoted as redeemable at 105 in 10 years is purchased to yield $j_2 = 10\%$. If this same bond was redeemable at par, the actual purchase price would be \$113.07 less. Determine the value of X. *Ans.* \$6000

8.47 A corporation issues a special 20-year bond issue that has no coupons. Rather, interest will accumulate on the bond at rate $j_2 = 11\%$ for the life of the bond. At the time of maturity, the total value of the loan will be paid off, including all accumulated interest. Find the price of a \$1000 bond of this issue to yield $j_2 = 10\%$. *Ans.* \$1209.28

8.48 A \$1000 bond bearing coupons at $j_2 = 13\%$ and redeemable at par is bought to yield $j_2 = 12\%$. If the present value of the redemption value at this yield is \$140, what is the purchase price?
Ans. \$1071.67

8.49 Two \$1000 bonds redeemable at par at the end of n years are bought to yield $j_2 = 10\%$. One bond costs \$1153.72 and has semiannual coupons at $j_2 = 12\%$. The other bond has semiannual coupons at $j_2 = 8\%$. Find the price of the second bond. *Ans.* \$846.28

8.50 A \$1000 bond with semiannual coupons at $j_2 = 12\%$ and redeemable at the end of n years at \$1050 sells at \$930 to yield $j_2 = 15\%$. Find the price of a \$1000 bond with semiannual coupons at $j_2 = 10\%$ redeemable at the end of $2n$ years at \$1040 to yield $j_2 = 15\%$. *Ans.* \$767.62

8.51 A bond with a par value of \$100 000 has coupons at the rate $j_2 = 13\%$. It will be redeemed at par when it matures a certain number of years hence. It is purchased for a price of \$94 702.99. At this price, the purchaser who holds the bond to maturity will realize a yield rate $j_2 = 14\%$. Find the number of years to maturity. *Ans.* 10

8.52 A \$1000 bond with annual coupons at 8.5% and maturing in 20 years at par is purchased to yield an annual effective rate of interest of 9% if held to maturity. The book value of the bond at any time is the discounted value of all remaining payments, using the 9% rate. Ten years later, just after a coupon payment, the bond is sold to yield the new purchaser a 10% annual effective rate of interest if held to maturity. Find the excess of the book value over the second sale price. *Ans.* \$60.08

8.53 A corporation has an issue of bonds with annual coupons at $j_1 = 10\%$ maturing in 5 years at par which are quoted at a price that yields $j_1 = 12\%$. (*a*) What is the price of a \$1000 bond? (*b*) It is proposed to replace this issue of bonds with an issue of bonds with annual coupons at $j_1 = 11\%$. How long must the new issue run so that the bond holders will still yield $j_1 = 12\%$? Express your answer to the nearest year. *Ans.* (*a*) \$927.90; (*b*) 18

8.54 A \$1000 bond bearing semiannual coupons at $j_2 = 10\%$ is redeemable at par. What is the minimum number of whole years that the bond should run so that a person paying \$1100 for it would earn at least $j_2 = 8\%$? *Ans.* 7

8.55 If the coupon rate on a bond is 1.5 times the yield rate when it sells for a premium of $10 per $100, find the price per $100 for a bond with the same number of coupons to run and the same yield with coupons equal to $\frac{3}{4}$ of the yield rate. *Ans.* $95

CALLABLE BONDS

8.56 A $5000 bond with semiannual coupons at $j_2 = 11\%$ is redeemable at par in 20 years; it is callable at par in 15 years. Find the price to guarantee a yield of (*a*) $j_2 = 13\%$; (*b*) $j_2 = 9\%$.
Ans. (*a*) $4292.72; (*b*) $5814.44

8.57 A $2000 bond paying interest at $j_2 = 10\%$ is redeemable at par in 20 years. It is callable at par in 15 years. Find the price to guarantee a yield rate of (*a*) $j_2 = 8\%$; (*b*) $j_2 = 12\%$.
Ans. (*a*) $2345.84; (*b*) $1699.07

8.58 A $1000 bond paying bond interest at $j_2 = 13\%$ is redeemable at par in 20 years or callable at 105 in 15 years. Find the price to guarantee a yield of (*a*) $j_2 = 15\%$, (*b*) $j_2 = 11\%$.
Ans. (*a*) $874.06; (*b*) $1160.46

8.59 A $2000 bond with semiannual coupons at 12% is redeemable at par in 20 years. It is callable after 10 years at 110, and after 15 years at 105. Find the price to guarantee a yield of $j_\infty = 11\%$.
Ans. $2108.81

8.60 A $2000 bond with semiannual coupons at $j_2 = 13\%$ is redeemable at par in 20 years. It is callable at a 5% premium in 15 years. Find the price to guarantee a yield rate of (*a*) $j_4 = 16\%$; (*b*) $j_1 = 11\%$. *Ans.* (*a*) $1610.79; (*b*) $2358.61

8.61 A $1000 bond with coupons at $j_2 = 11\%$ is redeemable at par in 20 years. It also has the following call options:

Call Date	Redemption
15 Years	105
16 years	104
17 years	103
18 years	102
19 years	101

Find the price to guarantee a yield rate of (*a*) $j_1 = 10\%$; (*b*) $j_{12} = 12\%$.
Ans. (*a*) $1107.63; (*b*) $903.75

PREMIUM AND DISCOUNT

8.62 For each bond in Table 8-7, find the purchase price and make out a complete bond schedule showing the amortization of the premium or accumulation of the discount.

Table 8-7

	Face Value	Redemption	Bond Interest Rate, j_2	Years to Redemption	Yield Rate
(*a*)	$500	at par	11%	3	$j_2 = 9\%$
(*b*)	10 000	at par	$10\frac{1}{2}\%$	$2\frac{1}{2}$	$j_2 = 12\%$
(*c*)	1 000	at par	12%	3	$j_4 = 10\%$
(*d*)	2 000	110	9 %	3	$j_{12} = 12\%$
(*e*)	1 000	105	12%	$2\frac{1}{2}$	$j_{365} = 14\%$
(*f*)	5 000	103	11%	$2\frac{1}{2}$	$j_2 = 10\%$

Ans. (*a*) $525.79; (*b*) $9684.07; (*c*) $1047.49; (*d*) $1978.09; (*e*) $984.35; (*f*) $5225.77

8.63 A \$1000-par-value bond, with semiannual coupons at 12% payable March 1 and September 1, has a book value of \$1075 on March 1, 1995, the underlying yield rate having been $j_2 = 10\%$. Find the amount for amortization of the premium on September 1, 1995, and the new book value at that time. *Ans.* \$6.25, \$1068.75

8.64 A 20-year bond with semiannual coupons is bought at a premium to yield $j_2 = 12\%$. If the amount of amortization of the premium at the time of the 4th coupon is \$4, determine the amount of amortization of the premium at the time of the 15th coupon. *Ans.* \$7.59

8.65 A \$1000 bond, redeemable at par, with annual coupons at 13%, is purchased for \$1184.34. If the write-down in the book value is \$11.57 at the end of the first year, what is the write-down at the end of the fifth year? *Ans.* \$16.94

8.66 A bond with \$65 semiannual coupons is purchased to yield $j_2 = 12\%$. If the first write-down is \$0.87, find the purchase price of the bond. *Ans.* \$1068.83

8.67 A \$1000 bond, redeemable at par, with annual coupons at 10% is purchased for \$1060. If the write-down in the book value is \$7 at the end of the first year, what is the write-down at the end of the 4th year? *Ans.* \$9.01

8.68 A bond with \$160 annual coupons is purchased at a discount to yield $j_1 = 15\%$. The write-up for the first year is \$44. What was the purchase price? *Ans.* \$1360

8.69 A \$1000 bond redeemable at \$1050 on December 1, 1998, pays interest at $j_2 = 13\%$. The bond is bought on June 1, 1996. Find the price and construct a bond schedule if the desired yield is (*a*) $j_{12} = 12\%$; (*b*) $j_1 = 11\%$. *Ans.* (*a*) \$1051.69; (*b*) \$1087.54

8.70 A \$1000, 20-year par value bond with semiannual coupons is bought at a discount to yield $j_2 = 10\%$. If the amount of the write-up of the discount in the last entry in the schedule is \$5, find the purchase price of the bond. *Ans.* \$909.91

8.71 A \$1000 bond pays coupons at $j_2 = 14\%$ on January 1 and July 1 and will be redeemed at par on July 1, 2000. If the bond was bought on January 1, 1992, to yield 12% per annum compounded semiannually, find the interest due on the book value on January 1, 1996. *Ans.* \$64.42

8.72 A \$1000 bond providing annual coupons at $j_1 = 9\%$ is redeemable at par on November 1, 2000. The write-down in the first year was \$5.63. The write-down in the eleventh year was \$19.08. Determine the book value of the bond on November 1, 1996. *Ans.* \$881.53

8.73 A \$2000 bond with annual coupons matures at par in 5 years. The first interest coupon is \$400, with subsequent coupons reduced by 25% of the previous year's coupon, each year. (*a*) Find the price to yield $j_1 = 10\%$. (*b*) Draw up the bond schedule. *Ans.* (*a*) \$2216.30

8.74 A 10-year bond matures for \$2000 and has annual coupons. The first coupon is \$100, and each increases by 10%. The bond is priced to yield $j_1 = 9\%$. Find the price and draw up the bond schedule. *Ans.* \$1801.07

8.75 A \$1000 bond with semiannual coupons at $j_2 = 10\%$ is redeemable for \$1100. If the amount for the 16th write-up is \$5, calculate the purchase price to yield $j_2 = 12\%$. *Ans.* \$868.11

8.76 You are told that a \$1000 bond with semiannual coupons at $j_2 = 8\%$ redeemable at par will be sold at \$700 to an investor requiring 12% per annum compounded semiannually. (*a*) Find the price of this bond to the same investor if the above coupon rate were changed to $j_2 = 11\%$. (*b*) Is the 11% bond purchased at a premium or a discount? (*c*) For the 11% bond show the write-up (or write-down) entries at the first two coupon dates.
Ans. (*a*) \$925; (*b*) Discount; (*c*) \$0.50, \$0.53

8.77 A \$10 000, 15-year bond is priced to yield $j_4 = 12\%$. It has quarterly coupons of \$200 each the first year, \$215 each the second year, \$230 each the third year, ..., \$410 each the fifteenth year. (*a*) Show that the price is \$9267.05. (*b*) Find the book value after 14 years. (*c*) Draw up a partial bond schedule showing the first and last year's entries only.
Ans. (*b*) \$10 408.88

PRICE OF A BOND BETWEEN BOND INTEREST DATES

8.78 Find the purchase price of the bonds of Table 8-8.

Table 8-8

	Face Value	Bond Interest Rate, j_2	Yield Rate	Redemption	Date of Purchase
(a)	\$500	$9\frac{1}{2}\%$	$j_2 = 11\%$	Jan. 1, 2013, at par	May 8, 1995
(b)	1000	11%	$j_2 = 10\%$	Jan. 1, 2008, at par	Oct. 3, 1994
(c)	2000	10%	$j_2 = 14\%$	Nov. 1, 2004, at par	July 20, 1995
(d)	1000	12%	$j_4 = 10\%$	Feb. 1, 2013, at par	Oct. 27, 1995
(e)	5000	$10\frac{1}{2}\%$	$j_{12} = 12\%$	July 1, 2004, at 105	July 30, 1994
(f)	2000	13%	$j_2 = 10\%$	Oct. 1, 2009, at 110	Apr. 17, 1995

Ans. (a) \$438.62; (b) \$1100.63; (c) \$1472.05; (d) \$1179.89; (e) \$4609.03; (f) \$2502.82

8.79 Find the purchase price of each \$1000 bond in Table 8-9 if bought at the given market quotation.

Table 8-9

	Redemption Value	Bond Interest Rate, j_2	Market Quotation	Redemption Date	Date of Purchase
(a)	par	11%	$101\frac{7}{8}$	Sept. 1, 2002	June 8, 1995
(b)	par	$10\frac{1}{2}\%$	$96\frac{1}{2}$	Feb. 1, 1998	Oct. 2, 1995
(c)	\$1050	9%	$92\frac{1}{8}$	Oct. 1, 1999	Nov. 29, 1996
(d)	1100	13%	$104\frac{1}{4}$	Apr. 1, 1998	Jan. 12, 1995

Ans. (a) \$1048.34; (b) \$982.69; (c) \$935.76; (d) \$1079.08

FINDING THE YIELD RATE

8.88 Find the yield rate of each bond in Table 8-11 by the Method of Averages.

Table 8-11

	Face Value	Redemption Value	Bond Interest Rate, j_2	Years to Redemption	Purchase Price
(*a*)	\$2000	at par	$10\frac{1}{2}\%$	12	\$1920
(*b*)	1000	at par	9%	10	910
(*c*)	5000	at 105	13%	$8\frac{1}{2}$	5860
(*d*)	500	at 110	11%	15	580

Ans. (*a*) 11.05%; (*b*) 10.37%; (*c*) 10.41%; (*d*) 9.38%

8.89 Redo Problem 8.88, using the Method of Interpolation.
Ans. (*a*) 11.12%; (*b*) 10.48%; (*c*) 10.30%; (*d*) 9.33%

8.90 A \$1000 bond with semiannual coupons at 8% matures at par on September 1, 2002. If this bond was quoted at 76 on September 1, 1995, what was the yield rate, j_2, (*a*) using the Method of Averages? (*b*) Using the Method of Interpolation? *Ans.* (*a*) 12.99%; (*b*) 13.40%

8.91 A \$1000 bond, redeemable at par at the end of 20 years and paying bond interest at $j_2 = 10\%$, is callable in 15 years at \$1050. Find the yield rate, j_2, if the bond is quoted now at $92\frac{3}{4}$ and (*a*) it is called, (*b*) it is not called. *Ans.* (*a*) 11.15%; (*b*) 10.90%

8.92 A \$2000 bond with semiannual coupons at 11% matures at par on June 1, 2011. On August 17, 1995, this bond is quoted at $96\frac{1}{2}$. What is the yield rate (*a*) j_2? (*b*) j_∞?
Ans. (*a*) 11.50%; (*b*) 11.18%

8.93 The ABC Corporation has a \$1000 bond that pays bond interest at $j_2 = 12\frac{1}{2}\%$. The bond is redeemable at par on September 1, 2003. On September 1, 1987, an investor buys this bond on the open market at 96; on September 1, 1995, he sells the bond on the open market at 106. Find his yield rate (*a*) j_2, (*b*) j_1. *Ans.* (*a*) 13.78%; (*b*) 14.25%

8.94 Ms. Holman buys a \$1000 bond that pays bond interest at $j_2 = 11\%$ and is redeemable at par in 15 years. The price she pays will give her a yield of $j_2 = 12\%$ if held to maturity. After 5 years, Ms. Holman sells this bond to Mr. Dawson, who desires a yield of $j_2 = 10\%$ on his investment. (*a*) What price did Ms. Holman pay? (*b*) What price did Mr. Dawson pay? (*c*) What yield j_2 did Ms. Holman realize? *Ans.* (*a*) \$931.18; (*b*) \$1062.31; (*c*) 13.86%

8.95 The XYZ Corporation issues a \$1000 bond with semiannual coupons at $j_2 = 11\%$ redeemable at par in 20 years or callable at par in 15 years. Mr. LaBelle buys the bond to guarantee a yield rate $j_{12} = 12\%$. (*a*) Find the purchase price. (*b*) After 15 years, the XYZ Corporation calls the bond in and pays Mr. LaBelle his \$1000. Find his overall yield stated as a rate j_{12}.
Ans. (*a*) \$903.75; (*b*) 12.13%

8.96 A corporation has a bond due December 4, 2008, paying bond interest at $j_2 = 10\frac{3}{8}\%$. The price bid on December 14, 1995, is 104. What yield j_{12} does the investor desire? *Ans.* 9.63%

8.97 Mr. Hunter buys a \$1000 bond with semiannual coupons at $j_2 = 12\%$. The bond is redeemable at par in 20 years. The price he pays will guarantee him a yield of $j_4 = 16\%$ if held to maturity. After 5 years, Mr. Hunter sells this bond to Miss Schnarr, who desires a yield of $j_1 = 11\%$ on her investment. (*a*) What price did Mr. Hunter pay? (*b*) What price did Miss Schnarr pay? (*c*) What yield, j_4, did Mr. Hunter realize? *Ans.* (*a*) \$746.78; (*b*) \$1095.02; (*c*) 21.13%

8.98 The ABC Corporation issues a \$1000 bond, with coupons payable at $j_2 = 11\%$ on January 1 and July 1, redeemable at par on July 1, 2005. (*a*) How much would an investor pay for this bond on September 1, 1995, to yield $j_2 = 13\%$? (*b*) Given the purchase price from part (*a*), if each coupon is deposited in a bank account paying interest at $j_4 = 10\%$ and the bond is held to maturity, what is the effective annual yield rate j_1 on this investment?
Ans. (*a*) \$909.30; (*b*) 12.24%

8.99 A bond paying semiannual coupons at $j_2 = 14\%$ matures at par in n years and is quoted at 110 to yield rate j_2. A bond paying semiannual coupons at $j_2 = 14\frac{1}{2}\%$ matures at par in n years and is quoted at 112 on the same yield basis. (*a*) Find the unknown yield rate, j_2. (*b*) Determine n to the nearest half-year. *Ans.* (*a*) 11.5%; (*b*) $5\frac{1}{2}$ years

8.100 An issue of bonds, redeemable at par in n years, is to bear coupons at $j_2 = 9\%$. An investor offers to buy the entire issue at a premium of 15%. At the same time, she advises the issuer that if the coupon rate were raised from $j_2 = 9\%$ to $j_2 = 10\%$, she would offer to buy the entire issue at a premium of 25%. At what yield rate j_2 are these two offers equivalent? *Ans.* $7\frac{1}{2}\%$

8.101 In August, 1991, interest rates in the bond markets were such that an investor could expect a yield of $j_2 = 10\%$. On August 1, 1991, an investor bought a bond with semiannual coupons at $j_2 = 9\%$ which was to mature in 20 years at par. By August 1, 1992, interest rates had fallen to $j_2 = 8\%$. This same investor sold his bond on August 1, 1992, on the bond market. Find the yield j_2 on this investment. *Ans.* 28.50%

OTHER TYPES OF BONDS

8.102 A \$10 000 serial bond is to be redeemed in instalments of \$2000 at the end of each of the 21st through 25th year from the date of issue. The bonds pay interest at $j_2 = 11\%$. What is the price to yield 9% per annum compounded semiannually? *Ans.* \$11 926.56

8.103 A \$100 000 issue of serial bonds issued July 1, 1995, and paying interest at rate $j_2 = 9\%$ is to be redeemed in instalments of \$25 000 each on the first day of July in 2000, 2002, 2004, and 2006. Find the price to yield 7% per annum compounded semiannually. *Ans.* \$111 898.48

8.104 Redo Problems 8.102 and 8.103 using Makeham's formula (Problem 8.33).

8.105 To finance an expansion of production capacity, Minicorp Ltd. will issue, on March 15, 1996, \$30 000 000 of serial bonds. Bond interest at 13% per annum is payable half yearly on March 15 and September 15, and the contract provides for redemption as follows:

\$10 000 000 of the issue to be redeemed March 15, 2001;
\$10 000 000 of the issue to be redeemed March 15, 2006;
\$10 000 000 of the issue to be redeemed March 15, 2011.

Calculate the purchase price of the issue to the public to yield $j_1 = 12\%$ on those bonds redeemable in 5 years and $j_1 = 14\%$ on the remaining bonds. *Ans.* \$29 861 132.81

8.106 A \$210 000 issue of serial bonds with annual coupons of 10% on the balance outstanding is to be redeemed in 20 annual instalments beginning at the end of 1 year. The amount redeemed at the end of 1 year is to be \$1000; at the end of 2 years, \$2000; at the end of 3 years, \$3000; and so on, so that the last redemption amount is \$20 000 at the end of 20 years. Find the price to be paid for this issue to yield $j_1 = 8\%$. [Note that Makeham's formula may prove to be useful.]
Ans. \$242 773.02

8.107 A \$10 000 serial bond is to be redeemed in \$1000 instalments of principal per year over the next 10 years. Coupons at the rate of 9% on the outstanding balance over the next 10 years are also to be paid annually. Calculate the purchase price to yield an investor a yield (*a*) $j_1 = 10\%$; (*b*) $j_2 = 8\%$. *Ans.* (*a*) \$9614.46; (*b*) \$10 343.62

8.80 What would be the market quotations on the $1000 bonds of Table 8-10?

Table 8-10

	Redemption Value	Bond Interest Rate, j_2	Yield Rate	Redemption Date	Date of Purchase
(*a*)	par	9%	$j_2 = 12\%$	Nov. 1, 2012	Feb. 8, 1995
(*b*)	par	13%	$j_2 = 10\%$	Mar. 1, 2014	Aug. 19, 1997
(*c*)	$1050	$10\frac{1}{2}\%$	$j_4 = 12\%$	June 1, 2008	Oct. 30, 1995
(*d*)	1100	11%	$j_2 = 9\%$	Oct. 1, 2006	Nov. 2, 1996

Ans. (*a*) $78\frac{1}{8}$; (*b*) 124; (*c*) $90\frac{1}{2}$; (*d*) $117\frac{1}{8}$

8.81 A $1000 bond, redeemable at par on October 1, 1997, is paying semiannual coupons at $j_2 = 10\%$. Find the purchase price on August 7, 1995 to yield $j_2 = 13\%$. Draw a diagram similar to Fig. 8-3 for this bond. *Ans.* $980.30

8.82 A $1000 bond, redeemable at $1100 on November 7, 2004, has coupons at $j_2 = 11\%$. Find the purchase price on April 18, 1995, if the desired yield is (*a*) $j_{12} = 13\%$; (*b*) $j_1 = 9\%$.
Ans. (*a*) $953.16; (*b*) $1232.88

8.83 An investment company is being audited and must locate the complete records of a specific bond transaction. The bond was a $500 bond with bond interest at rate $j_2 = 12\%$ redeemable at par on January 1, 2010. It was purchased for $550.89 sometime between July 1, 1995, and January 1, 1996. Find the exact date if the bond was purchased to yield $j_2 = 11\%$.
Ans. October 3, 1995

8.84 A National Auto Company Limited $1000 bond is due at par on December 1, 2006. Interest is payable at $j_2 = 12\%$ on June 1 and December 1. The bond may be called at 104 on December 1, 2000. Find the flat (purchase) price and the market price for this bond on August 8, 1995 if the yield is to be $j_2 = 10.5\%$, (*a*) assuming the bond is called at December 1, 2000; (*b*) assuming the bond matures at par on December 1, 2006. *Ans.* (*a*) $1083.12; (*b*) $1097.96

8.85 A $5000 bond with semiannual coupons at $j_2 = 9\%$ is redeemable at par on November 1, 2013. (*a*) Find the price on November 1, 1993 to yield $j_4 = 10\%$. (*b*) Find the book value of the bond on May 1, 1996, (just after the coupon is cashed). (*c*) What should the market quotation of this bond be on August 17, 1996 if the buyer wants a yield of $j_1 = 7\%$.
Ans. (*a*) $4521.50; (*b*) $4543.09; (*c*) $121\frac{1}{8}$

8.86 (*a*) A $1000 bond paying interest at $j_2 = 10\%$ is redeemable at par on September 1, 2013. Find the price on its issue date of September 1, 1993 to yield $j_2 = 12\%$. (*b*) Find the book value of the bond on September 1, 1995, (just after the coupon is cashed). (*c*) Find the sale price of this bond on September 1, 1995, if the buyer wants a yield of (i) 9% compounded semiannually; (ii) 15% compounded semiannually. (*d*) What should the market quotation of this bond be on October 8, 1995 to yield a buyer $j_2 = 11\%$?
Ans. (*a*) $849.54; (*b*) $853.79; (*c*) (i) $1088.33; (ii) $691.34; (*d*) $92\frac{1}{4}$

8.87 The ABC Corporation $5000 bond that pays interest at $j_2 = 11\%$ matures at par on October 1, 2007. (*a*) What did the buyer pay for the bond if it was sold on July 28, 1995, at a market quotation of $89\frac{3}{8}$? (*b*) What should be the market quotation of this bond on July 28, 1997, to yield a buyer $j_{12} = 9\%$? (*c*) What should be the market quotation of this bond on December 13, 1997, to yield a buyer $j_1 = 12\%$? *Ans.* (*a*) $4646.07; (*b*) $113\frac{1}{4}$; (*c*) $96\frac{1}{4}$

8.108 A corporation issues a \$1000, 20-year bond with bond interest at $j_2 = 10\%$. It is purchased by an investor A who wishes a yield of $j_2 = 9.5\%$. This investor A keeps the coupons but sells the strip bond to another investor whose desired yield is $j_{12} = 10.5\%$. Determine the overall rate j_2 to investor A. *Ans.* 8.99%

8.109 A corporation issued a \$1000, 15-year bond with bond interest at $j_2 = 9.5\%$ and redeemable at 105. It is purchased by Investor A who wishes a yield of $j_2 = 10\%$. Investor A sells the coupons to Investor B whose desired yield is $j_{12} = 10.5\%$ and Investor A keeps the strip bond. Determine the overall yield rate j_2 to Investor A. *Ans.* 9.20%

8.110 An annuity bond offers to repay \$10 000 principal and interest at $j_2 = 9\%$ by equal payments at the end of each half-year for 10 years. How much will an investor offer for this bond if he wants to realize (*a*) $j_2 = 8\%$; (*b*) $j_2 = 10\%$? *Ans.* (*a*) \$10 447.70; (*b*) \$9580.45

8.111 An annuity bond promises to repay \$50 000 principal with interest at $j_2 = 7\%$ by 20 equal semiannual payments, first payment in 3 years. How much should an investor, who wants to yield $j_1 = 8\%$, pay for this bond (*a*) now; (*b*) in 2 years; (*c*) in 5 years?
Ans. (*a*) \$47 167.03; (*b*) \$55 015.62; (*c*) \$46 707.38