UNIT III

Annuities

An annuity is a series of payments of a fixed amount of money at regular intervals of time. Usually the interval is a year. But the interval may be half-year, quarter year or month, etc., unless otherwise stated about this interval, it will be taken as a year. The payments may be for a fixed number of years or to continue for ever. If it is for a fixed number of years then the annuity is called **Annuity certain.** If it is continued for ever it is called **Perpetual Annuity,** or **Perpetuity.** The annuity which is payable till the happening of an event is called **annuitant contingent.** Annuity till the marriage of a girl, the annuity till the death of a person are examples of annuity contingent.

If each payment of an annuity is made at the end of each period the annuity is called **Immediate annuity.** If each payment is made at the beginning of each period, the annuity is called **Annuity Due.**

When an annuity is payable after a lapse of a given period, it is called **Deferred Annuity.** When an annuity is deferred for r years the first payment is to be made at the end of $(r+1)$ year. The amount of an annuity for n years is the sum of all instalments with compound interest due at the end of n years. This is called the Accumulated sum of an annuity.

The **present value** of an annuity is the sum of all present values of various instalments of the annuity.

A sinking fund is a fixed amount of money deposited periodically which grows with compound interest in order to meet some future liability. Usually sinking funds are invested for the liquidation of a loan or a debenture stock, for replacing a machinery or for meeting the marriage expenses, etc.

Leasehold estate :

An estate yielding a fixed annual rent for a given number of years is called a lease-hold estate. After the expiry of the period it is reverted to the original owner.

Free-hold Estate :

If an estate is held for ever it is called a free-hold estate, and it yields a perpetual annuity, called rent.

Amortization :

When loans are repaid in equal periodical instalments, each instalment amount is split up into two components called repayment of principal and payment of interest on the outstanding balance. The principal components in these instalments are called Amortization.

The types of problems on annuities are,

(1) to find the amount of an annuity for n years,

(2) to find the present value of an annuity for n years.

(3) to find the amount of each annual instalment to be paid for n years which will accrue to a given sum of money at the end of n years.

(4) to find the repayment principals and payment of interests in each instalment for paying off the loan.

We shall now derive formula for finding the amount of an immediate annuity and an annuity due.

Formula for Amount of Immediate annuity :

Let A be the amount of an annuity of Rs. a for n years and i be the rate of interest per rupee. Since the annuity is for n years and each instalment is paid at the end of the year, the 1st instalment will earn interest for (n−1) years, the 2nd instalment will earn interest for (n−2) years, and so on and the last instalment will earn no interest.

∴ The amount of 1st instalment $= a\,(1+i)^{n-1}$

The amount of 2nd instalment $= a\,(1+i)^{n-2}$

The amount of 3rd instalment $= a\,(1+i)^{n-3}$

The amount of last instalment $= a$

$$\therefore\ A = a\,(1+i)^{n-1} + a\,(1+i)^{n-2} + a\,(1+i)^{n-3} + \dots\dots\dots + a$$

$$= [\, a\,(1+i)+(1+i)^{n-2}+(1+i)^{n-3}+\dots\dots\dots\dots\dots+1\,]$$

$$= a\left[\frac{(1+i)^n-1}{(1+i)\;-1}\right]$$

$$= \frac{a}{i}\,[\,(1+i)^n-1\,] \qquad \dots \quad (1)$$

Formula for the amount of annuity due :

In this case the instalment money is paid at the beginning of the year.

The amount of first instalment after n years $= a\,(1+i)^n$

The amount of 2nd instalment $= a\,(1+i)^{n-1}$

....

∴ The amount of last instalment $= a\,(1+i)$

∴ The amount of annuity due is

..

$$A = a(1+i)^n + a(1+i)^{n-1} + a(1+i)^{n-2} + \dots\dots\dots + a(1+i)$$

$$= a\left[(1+i)^n + (1+i)^{n-1} + (1+i)^{n-2} + \dots\dots\dots (1+i)\right]$$

$$= a\,(1+i) \left[\frac{(1+i)^n - 1}{(1+i) - 1} \right]$$

$$= \frac{a\,(1+i)}{i} \left[(1+i)^n - 1 \right]$$

Note 1 :

If the instalment amount per year is split up and paid k times a year, then

$$A = \frac{a/k}{i/k} \left[(1+i)^{nk} - 1 \right]$$

$$= \frac{a}{i} \left[(1+i)^{nk} - 1 \right] \qquad \dots \quad (2)$$

Note 2 :

If the instalment amount is paid once in a year and the interest is compounded k times a year, then

$$A = a\left(1 + \frac{i}{k}\right)^{(n-1)k} + a\left(1 + \frac{i}{k}\right)^{(n-2)k} + \dots\dots + a$$

$$= \frac{a\left[\left(1 + \frac{i}{k}\right)^{nk} - 1\right]}{\left(1 + \frac{i}{k}\right)^{k} - 1} \qquad \dots \quad (3)$$

When k = 1, this is the same as (1).

Present Value of an immediate annuity .

The present value of Rs a to be paid at the end of 1st year at the rate of i per rupee is equivalent to Rs. $\frac{a}{1+i}$

The present value of Rs. a to be paid at the

end of 2nd year $= \frac{a}{(1+i)^2}$, at the

end of 3rd year $= \frac{a}{(1+i)^3}$, etc.

∴ The sum of all these present values is

$$P = \frac{a}{1+i} + \frac{a}{(1+i)^2} + \dots\dots + \frac{a}{(1+i)^n}$$

$$= \frac{a}{1+i}\left[1 + \frac{1}{1+i} + \dots\dots + \frac{a}{(1+i)^{n-1}}\right]$$

$$= \frac{a}{1+i}\left[\frac{1 - \frac{1}{(1+i)^n}}{1 - \frac{1}{1+i}}\right]$$

$$= \frac{a}{i}\left[1 - (1+i)^{-n}\right]$$

Note 1 :

If the annuity is the annuity due, then

$$P = \frac{a(1+i)}{i}\left[1 - \frac{1}{(1+i)^n}\right].$$

Note 2 :

In case of immediate annuity if the instalments are paid k times a year, then

$$P = \frac{a}{i}\left[1 - \frac{1}{\left(1 + \frac{i}{k}\right)^{nk}}\right]$$

Note 3 :

In the case of immediate annuity of the instalment amount a is split up into k equal amounts and paid k times a year, then

$$P = \frac{a\left[1 - \frac{1}{\left(1+\frac{i}{k}\right)^{nk}}\right]}{1 - \frac{1}{\left(1+\frac{i}{k}\right)^{k}}}$$

$$= \frac{a\left[1 - \left(1+\frac{i}{k}\right)^{-nk}\right]}{1 - \left(1+\frac{i}{k}\right)^{-k}}$$

Similar results can also be obtained in the case of annuity due.

Present value of Deferred annuity :

The 1st instalment is paid only after $d+1$ years,

$$P = \frac{a}{(1+i)^{d+1}} + \frac{a}{(1+i)^{d+2}} + \ldots\ldots\ldots\ldots\ldots \text{ to n terms}$$

$$= \frac{a}{(1+i)^{d+1}} \frac{\left[1 - \frac{1}{(1+i)^{n}}\right]}{1 - \frac{1}{1+i}}$$

$$= \frac{a}{(1+i)^{d}} \cdot \frac{1}{i} \left[1 - (1+i)^{-n}\right]$$

Present value of a perpetuity :

Let a be the amount of each annual instalment of a perpetual annuity. Then the present value of the perpetual annuity is,

$$P = \frac{a}{1+i} + \frac{a}{(1+i)^{2}} + \frac{a}{(1+i)^{3}} + \ldots\ldots\ldots\ldots\ldots\ldots \infty$$

$$= \frac{\frac{a}{1+i}}{1 - \frac{a}{1+i}} = \frac{a}{1}$$

Note :

The present value of a perpetuity deferred for *d* years

$$= \frac{1}{(1+i)^d} \times \frac{1}{i}$$

Example 1 :

Find the amount of an annuity of Rs. 2,000 per annum for 10 years reckoning compound interest at 10% per annum.

$a = 2000$

$i = 0{\cdot}1$

$n = 10$

$$A = \frac{a}{i} [(1+i)^n - 1]$$

$$= \frac{2000}{0.1} [(1+{\cdot}1)^{10} - 1]$$

$$= 20{,}000 [(1{\cdot}1)^{10} - 1]$$

$$\log (1{\cdot}1)^{10} = 10 \times {\cdot}0414 = {\cdot}414$$

$$\therefore \quad (1{\cdot}1)^{10} = 2{\cdot}594$$

$$\therefore \quad A = 20{,}000 (2{\cdot}594 - 1)$$

$$= 20{,}000 \times 1{\cdot}594$$

$$= \text{Rs. } 31{,}880.$$

Example 2 :

A man wishes to have Rs. 2,500 available in a bank account when his daughter's first year college expenses begin. How much must he deposit in the beginning of each year at 3·5 per cent compounded annually, if the girl is to start in college six years from now ?

$A = 2{,}500$

$n = 6$ years

$i = \cdot 035 \qquad a = ?$

Here A is the amount of annuity due for 6 years

$$A = \frac{a\,(1+i)}{i}\,\left(\,(1+i)^n - 1\,\right)$$

$$2500 = \frac{a\,(1 \cdot 035)}{\cdot 035}\,[\,(1 + \cdot 035)^6 - 1\,]$$

$$a = \frac{2500 \times \cdot 035}{1.035\,[\,(1 \cdot 035)^6 - 1\,]}$$

$$\log\,(1 \cdot 035)^6 = 6 \times 0 \cdot 0149 = 0 \cdot 0894$$

$$\therefore\ (1 \cdot 035)^6 = \text{antilog}\ 0 \cdot 0894$$

$$= 1 \cdot 228$$

$$\therefore\quad a = \frac{2500 \times \cdot 035}{1 \cdot 035 \times 0 \cdot 228}$$

$$= \text{Rs. } 370 \cdot 90 \text{ (using log tables)}$$

Example 3 :

On his 48th birthday, a man decides to make a gift of Rs. 5,000 to a hospital on his 60th birthday. He decides to save this amount by making equal annual payment upto and including his 60th birthday to a fund which gives $3\frac{1}{2}\%$ compound interest. The first payment being made at once. Calculate the amount of each annual payment.

$A =$ Rs. 5,000

number of instalments $= 13$

$I = \cdot 035$

The first instalment is being paid on his 48th birthday and the last one is being paid on his 60th birthday.

$$A = a(1+i)^{12} + a(1+i)^{11} + \ldots\ldots\ldots\ldots + a$$

$$= \frac{a}{i}\left[(1+i)^{13}-1\right]$$

$$5000 = \frac{a}{\cdot 035}\left[(1\cdot 035)^{13}-1\right]$$

$$a = \frac{5000 \times \cdot 035}{(1\cdot 035)^{13}-1}$$

$$= \text{Rs. } 310.30$$

Example 4 :

The accumulations in a provident fund are invested at the end of every year to earn 10% p.a. A person contributed $12\frac{1}{2}\%$ of his salary to which the employer adds 10% every month. Find how much the accumulations will amount to at the end of 30 years of his service, for every 100 rupees of his monthly salary. (Give the answer to the nearest rupee).

Self contribution to PF per month $= 12\frac{1}{2}\%$

Employee's contribution to the PF per month $= 10\%$

Total contribution to the PF per month $= 22\cdot 5\%$

Total contribution per year $= 22\cdot 5\% \times 12$

$= 270\%$

This amount is paid at the end of each year for every hundred rupees of his monthly salary.

Amount accumulated at the end of 30 years of service is

$$A = \frac{a}{i}\left[(1+i)^{n}-1\right]$$

$$= \frac{270}{0.1} \; [\, (1 \cdot 1)^{30} - 1 \,]$$

$$= 2700 \; [\, (1 \cdot 1)^{30} - 1 \,]$$

$$\log\,(1 \cdot 1)^{30} = 30 \times 0 \cdot 0414 = 1 \cdot 242$$

$$\therefore \quad (1 \cdot 1)^{30} = \text{antilog } 1 \cdot 242 = 17 \cdot 46$$

$$\therefore \quad A = 2700 \; [17 \cdot 46 - 1]$$

$$= 2700 \; [16 \cdot 46]$$

$$= \text{Rs. } 44{,}442.$$

Example 5 :

Find the least number of years for which an annuity of Rs. 250 must run in order that the amount just exceeds Rs. 4,000 at 5% compounded annually.

Here $a = 250$

$A = 4000$

$i = \cdot 05$

$n = ?$

$$A = \frac{a}{i} \; [\, (1+i)^n - 1 \,]$$

$$4000 = \frac{250}{\cdot 05} \; [\, (1 \cdot 05)^n - 1 \,]$$

$$4000 = 5000 \; [\, (1 \cdot 05)^n - 1]$$

$$(1 \cdot 05)^n - 1 = 0 \cdot 8$$

$$(1 \cdot 05)^n = 1 \cdot 8$$

$$n \log 1 \cdot 05 = \log 1 \cdot 8$$

$$n = \frac{\log 1 \cdot 8}{\log 1 \cdot 05}$$

$$= \frac{0 \cdot 2553}{0 \cdot 0212}$$

$$= 12 \cdot 04 \text{ years}$$

∴ The least number of years in which the accumulated amount just exceeds Rs. 4,000 is 13.

Example 6 :

A sinking fund was formed by setting aside Rs. 1,000 at the end of the first year and then at the end of each of the following years an amount 10% more than that set aside at the end of the immediately previous year. Find the total amount of the fund at the end of ten years, reckoning interest at 5 per cent per annum compounded. (Give your answer correct to the nearest rupee).

Amount set aside for the sinking fund at the end of 1st year } = Rs. 1,000

Amount set aside at the end of 2nd year $= 1000 \left(\frac{110}{10}\right)$

$= 1000 \; (1 \cdot 1)$

Amount set aside at the end of 3rd year $= 1000 \; (1 \cdot 1)^2$, etc.

Rate of interest = 5% per annum

(ie) $i = \cdot 05$

Period of annuity = 10 years

Amount to the fund at the end of 10 years is

$$A = 1000\,(1 \cdot 05)^9 + 1000\,(1 \cdot 1)\,(1 \cdot 05)^8 + 1000\,(1 \cdot 1)^2\,(1 \cdot 05)^7 + \ldots\ldots\ldots\ldots + 1000\,(1 \cdot 1)^9$$

$$= 1000\,[\,(1 \cdot 05)^9 + (1 \cdot 1)\,(1 \cdot 05)^8 + (1 \cdot 1)^2\,(1 \cdot 05)^7 + \ldots\ldots\ldots + (1 \cdot 1)^9\,]$$

$$= 1000\,(1 \cdot 05)^9 \left[1 + \frac{1.1}{1 \cdot 05} + \left(\frac{1 \cdot 1}{1 \cdot 05}\right)^2 + \ldots + \left(\frac{1.1}{1 \cdot 05}\right)^9 \right]$$

$$= 1000\,(1 \cdot 05)^9 \; \frac{\left[\left(\frac{1 \cdot 1}{1 \cdot 05}\right)^{10} - 1 \right]}{\frac{1 \cdot 1}{1 \cdot 05} - 1}$$

$$= 1000\,(1 \cdot 05)^9 \left[\frac{1 \cdot 592 - 1}{1 \cdot 047 - 1}\right]$$

$$= 1000\,(1 \cdot 05)^9 \left(\frac{\cdot 592}{\cdot 047}\right)$$

= **Rs. 19,540.**

Example 7 :

A company set aside for a reserve fund the sum of Rs. 20,000 annually to enable it to pay off a debenture issue of Rs. 2,39.000 at the end of 10 years. Assuming that the reserve accumulates at 4% p.a. compound, find the surplus after paying off the debenture issue.

Amount set aside each year = Rs. 20,000

Period of the repayment = 10 years

Interest rate = 4% p.a.

(ie) $a = 20{,}000$

$n = 10$

$i = \cdot 04$

$\therefore$ Amount accumulated in the fund after 10 years is

$$A = \frac{a}{i} [(1+i)^n - 1]$$

$$= \frac{20,000}{\cdot 04} [(1 \cdot 04)^{10} - 1]$$

$$= 5,00,000 \; [1 \cdot 479 - 1]$$

$$= 5,00,000 \times \cdot 479$$

$$= \text{Rs. } 2,39,500.$$

$\therefore$ Value of debenture stock = Rs. 2,39,000

$\therefore$ Surplus after paying of debenture = Rs. 5CC.

Example 8 :

Find the present value of an annuity of Rs. 5,000 per annum for 12 years, the interest being 4% per annum compounded annually.

a = Rs. 5,000

n = 12 years

i = ·04

$$P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n} \right]$$

$$= \frac{5000}{.04}\left[1 - \frac{1}{(1 \cdot 04)^{12}} \right]$$

$$= 1,25,000 \; [1 - 0 \cdot 6246]$$

$$= 1,25,000 \; (\cdot 3754)$$

$$= \text{Rs. } 46,925.$$

Example 9 :

A man retires at the age of 60 and earns a pension of Rs. 8,700 a year. He wants to commute one third of his pension. Find the amount he will receive, if the expecations of life at this age be 10 years, and the interest is compounded at 4% per annum. (Assume that pension for a year is due at the end of the year).

Annual pension = Rs. 8,700

$\frac{1}{3}$rd of the annual pension = Rs. 2,900

Commuted amount = $\frac{1}{3}$ of the pension

Here a = Rs. 2,900

n = 10 years

i = ·04

$$P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

$$= \frac{2900}{\cdot 04}\left[1 - \frac{1}{(1\cdot 04)^{10}}\right]$$

$$= 2900 \times 25\ [\,1 - \cdot 6756\,]$$

$$= 2900 \times 25 \times 0\cdot 3244$$

= Rs. 23,519

Example 10 :

A fixed royalty of Rs. 1,500 for 20 years is guaranteed to the author of some text books by a publisher. The right of receiving the royalty is put up for auction after 12 years have already been passed. Find to the nearest rupee, the price for which it may be sold, reckon compounded interest at 8%

Amount of royalty per year = Rs. 1,500

Period of royalty = 20 years

No. of years royalty received = 12 years

Remaining years for eligibility of royalty = 8 years

We have to find the present value of the royalty for 8 years at compound interest at the rate of 8% p.a.

(ie) a = Rs. 1,500

n = 8

i = ·08

P = ?

$$P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^8}\right]$$

$$= \frac{1500}{\cdot 08}\left[1 - \frac{1}{(1\cdot 08)^8}\right]$$

= Rs. 8,620

Example 11 :

A house is purchased on instalment basis such that Rs. 10,000 is to be paid on signing of the agreement and 10 yearly equal instalments of Rs. 5,000 each, the first to be paid one year after the date of purchase. If compound interest is charged at 5% p.a. what is the cash price of the house?

Instalment amount = Rs. 5000

i = ·05

n = 10

The present value of the instalment is

$$P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

$$= \frac{5000}{.05}\left[1 - \frac{1}{(1\cdot 05)^{10}}\right]$$

$$= 1,00,000\ [1 - \cdot 6138]$$

$$= 1,00,000 \times 0\cdot 3862$$

$$= \text{Rs. } 38,620$$

Initial payment = Rs. 10,000

∴ Cash price of the house = 10,000 + 38,620

= Rs. 48,620.

Example 12 :

A person desires to create an endowment fund to provide for a prize of Rs. 300 every year. If the fund can be invested at 10% p. a. compound interest, find the amount of the endowment.

Since the endowment is to continue for ever, we have to find the present value of a perpetual annuity.

a = Rs. 300

i = 0·1

$$r = \frac{A}{i} = \frac{300}{\cdot 1} = \text{Rs. } 3,000$$

∴ The amount of the endowment is Rs. 3,000.

Example 13 :

A loan of Rs. 1,000 is repayable in 5 equal annual instalments, compound interest is charged at 6% per annum and the first payment is made after a year. Analyse the payment into interest and amortization of principal.

(B.Com. Oct–1985).

First we have to find the instalment amount.

$$P = 1000$$

$$n = 5$$

$$i = \cdot 06$$

$$a = ?$$

$$P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

$$\text{(i.e)} \quad 1000 = \frac{a}{\cdot 06}\left[1 - \frac{1}{(1 \cdot 06)^5}\right]$$

$$= \frac{a}{\cdot 06}\,[\,1 - 0 \cdot 7473\,]$$

$$= \frac{a}{\cdot 06}\,(\cdot 2527)$$

$$\therefore \quad a = \frac{1000 \times \cdot 06}{\cdot 2527}$$

$$= \text{Rs. } 237 \cdot 39$$

Amortization of Principal

End of year	*a*	*interest*	*Principal*	*Principal earning interst*
1	237.39	60.00	177.39	1000.00
2	237.39	49.36	188.03	822.61
3	237.39	38.07	199.32	634.58
4	237.39	26.12	211.27	435.26
5	237.39	13.40	223.99	223.99
Total	1186.95	186.95	1000.00	

Example 14 :

At the age of 55, a person invests a certain sum of money at 10% compound interest per annum. This investment is just sufficient for him to receive Rs. 2,000 per year, for a period of 20 years. If the first payment to be received is due at the age of 61, find the amount invested.

The amount is invested at the age of 55 and he has to receive annual instalments from the age of 61. The problem is to find the present value of a deferred annuity for 5 years.

Here $a = \text{Rs. } 2{,}000$

$n = 20$

$d = 5$

$i = 0{\cdot}1$

$$P = \frac{1}{(1+i)^d} \cdot \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

$$= \frac{1}{(1{\cdot}1)^5} \cdot \frac{2000}{0{\cdot}1}\left[1 - \frac{1}{(1{\cdot}1)^{20}}\right]$$

$$= \frac{20{,}000}{(1{\cdot}1)^5}\,[\,1 - {\cdot}1486\,]$$

$$= \frac{20{,}000\,(0{\cdot}8514)}{(1{\cdot}1)^5}$$

$$= \text{Rs. } 10{,}573.$$

Example 15 :

A person deposits his whole fortune of Rs. 20,000 in a bank at 5% compound interest p.a. and settles to withdraw Rs. 1,800 per year for his personal expenses. If he begins to spend from the end of the first year and goes on spending at this rate, show that he will be ruined before the end of 17th year,

$P = 20{,}000$

$a = 1{,}800$

$i = \cdot 05$

$$P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

$$20{,}000 = \frac{1800}{.05}\left[1 - \frac{1}{(1\cdot 05)^n}\right]$$

$$1 - \frac{1}{(1\cdot 05)^n} = \frac{20{,}000 \times \cdot 05}{1{,}800}$$

$$\frac{1}{(1\cdot 05)^n} = \frac{4}{9}$$

$$n \log (1\cdot 05) = \log 9 - \log 4$$

$$n = \frac{\log 9 - \log 4}{\log 1\cdot 05}$$

$$= \frac{0\cdot 9542 - 0\cdot 6021}{0\cdot 012}$$

$$= \frac{0\cdot 3521}{\cdot 0212} = 16\cdot 6 \text{ years}$$

∴ He will be ruined before the end of 17th year.

EXERCISE 10

(1) Find the amount of an annuity of Rs. 1,000 in 10 years allowing compound interest at $4\frac{1}{2}\%$

(2) Find the amount of an annuity of Rs. 100 in 12 years allowing compound interest at 6% per annum.

(3) A person invests Rs. 1,000 every year with a company which pays interest at 10% per annum. He allows his deposits to accumulate with the company at compound interest.. Find the amount standing to his credit one year after he has made his yearly investment for the tenth time.

(4) Find the sum of money received by a pensioner at 58 if he wants to commute his annual pension of Rs. 1,200 for a present payment when compound interest is reckoned at 4% p.a., and the expectations of his life is assessed at 10 years only.

(5) A wagon is purchased on instalment basis, such that Rs. 5,000 is to be paid on signing the contract and balance in four yearly instalments of Rs. 3,000 each payable at the end of 1st, 2nd, 3rd and 4th year. If interest is charged at 5% p.a., what should be the cash down price?

(6) Find the present value of an annuity certain of Rs. 150 for 12 years interest at 5% per annum

(7) Calculate the present value of an annuity of Rs. 5,000 per annum for 12 years, the interest being 4% per annum compounded annually.

(8) A man buys an old piano for Rs. 500 agreeing to pay Rs. 100 down and the balance in equal monthly instalment of Rs. 20 with interest at 6%. How long will it take him to complete payment ?

(9) A company borrows Rs. 10,000 on condition to repay it with compound interest at 5% p. a. by annual instalments of Rs. 1,000 each. In how many years will the debt be paid off ?

(10) A person borrows a sum of Rs. 5,000 at 4% compound intesest. If the principal and interst are to be repaid in 10 equal annual instalments find the amount of each instalment, the first payment being made after 1 year.

(11) To accumulate a fund for his son's higher education a person invests a sum of Rs. 100 on the son's first birth day and an equal amount on each of subsequent birth days. If the interest is compounded half-yearly at the rate of 6% per annum, find the amount accumulated just after the investment has been made on the 18th birthday.

(12) A man wishes to pay back his debt of Rs. 2,522 due after 3 years by three equal yearly instalments. Find the amount of each instalment, money being worth 5% per annum compound interest.

(13) A machine costs a company Rs. 80,000 and its effective life is estimated to be 20 years. A sinking Find is created for replacing the machine at the end of its life time when its scrap realises a sum of rupees five thousand. Calculate to the nearest hundreds of rupees, the amount which should be provided every year for the sinking fund if it accumulates at 9% p.a. compound interest annually.

(14) A machine costs a company Rs. 1,00,000 and its effective life is estimated to be 20 years. If the scrap is expected to realise Rs. 5,000 only, find correct to the nearest rupee the sum to be invested every year at 5% per annum, compound interest for 20 years to replace the machine which is expected to cost then 25% more over its present cost.

(15) A company sets aside for a reserve fund the sum of Rs. 50,000 annually to enable it to pay off a debenture issue of Rs. 5,90,000 at the end of 10 years. Assuming that the reserve accumulates at 5% p. a. compound, find the surplus after paying off the debenture stock.

(16) A person retires at the age of 58 and earns a pension of Rs. 6,000 a year. He wants to commute one-fourth of his pension to ready-money. If the expectation of life at this age be 12 years, find the amount he will receive when money is worth 4% p.a. compound, if it is assumed that pension for a year is due at the end of the year.

(17) A man retires, at the age of 60 and his employer gives him a pension of Rs. 1,200 a year paid in half-yearly instalments for the rest of his life. Reckoning his expectation of life to be 13 years and that interest is at 4% p.a payable half-yearly what single sum is equivalent to his pension?

(18) A company borrows Rs. 10,000 on condition to repay it with compound interest at 5% per annum by annual instalments of Rs. 1,000 each. In how many years will the debt be paid off?

(19) A man wishes to have Rs. 2,500, available in a bank account when his daughter's first year college expenses begin.

How much must he deposit now at 3·5 per cent compound annually, if the girl is to start in college six years from now?

(20) A man purchases a house and takes a mortgage on it for Rs. 60,000 to be paid off in 12 years by equal annual payments. If the interest rate is 5% compounded annually, what amount will be required to pay each year?

(21) A loan of Rs. 1,500 is to be paid in 6 annual instalments, interest being paid at 10% p.a. compounded. Analyse the payments into those on account of interest and on account of amortization of the principal.

(22) Sri Biswas bought a house, paying Rs. 20,000 down and Rs. 4,000 at the end of each year for 25 years. What ought to have been paid for the house if he had bought it cash down? Reckon interest at 5% p.a. compound.

(23) For endowing an annual scholarship of Rs. 10,000, Mr. X. wishes to take five equal annual contributions. The first award of the scholarship is to be made two years after the last of his contributions. What would be the value of each contribution, assuming interest at 5% p.a. compound interest?