



UNIT-III PARTIAL DIFFERENTIAL EQUATIONS Solution of Second Order Partial Differential Equations

Type

$$\text{RHS} = e^{ax+by} \phi(x, y)$$

Replace D by $D+a$

D' by $D'+b$

J. So find the PI of $(D^2 - 2DD' + D'^2)z = x^2 y^2 e^{x+y}$

Soln.:

$$\text{PI} = \frac{1}{D^2 - 2DD' + D'^2} x^2 y^2 e^{x+y}$$

$$= \frac{1}{(D-D')^2} e^{x+y} x^2 y^2$$



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$$\begin{aligned}
 &= e^{x+y} \frac{1}{(D+1-(D'+1))^2} x^2 y^2 && D \rightarrow D+a=D+1 \\
 & && D' \rightarrow D'+b=D'+1 \\
 &= e^{x+y} \frac{1}{(D-D')^2} x^2 y^2 \\
 &= e^{x+y} \frac{1}{D^2 - 2DD' + D'^2} x^2 y^2 \\
 &= e^{x+y} \frac{1}{D^2 \left[1 - \frac{2D'}{D} + \frac{D'^2}{D^2} \right]} x^2 y^2 \\
 &= e^{x+y} \frac{1}{D^2} \left[1 - \left(\frac{2D'}{D} - \frac{D'^2}{D^2} \right) \right]^{-1} x^2 y^2 \\
 &= e^{x+y} \frac{1}{D^2} \left[1 + \frac{2D'}{D} - \frac{D'^2}{D^2} + \frac{4D'^2}{D^2} \right] x^2 y^2 \\
 &= e^{x+y} \frac{1}{D^2} \left[x^2 y^2 + \frac{2D'}{D} x^2 y^2 + \frac{3D'^2}{D^2} x^2 y^2 \right] \\
 &= e^{x+y} \left[\frac{1}{D^2} x^2 y^2 + \frac{2}{D^3} 2x^2 y + \frac{3}{D^4} (2x^2) \right] \\
 &= e^{x+y} \left[\begin{aligned}
 &\frac{1}{D^2} x^2 y^2 \rightarrow \frac{x^3}{3} y^2 \\
 &\quad \quad \quad 2^{nd} \rightarrow \frac{x^4}{12} y^2 \\
 &\frac{1}{D^3} 4x^2 y \rightarrow 4 \frac{x^3}{3} y \\
 &\quad \quad \quad 2^{nd} \quad \quad \quad 3^{rd} \\
 &\frac{4x^4}{12} y \rightarrow \frac{4x^5}{60} y \\
 &\frac{1}{D^4} 6x^2 \rightarrow \frac{6x^3}{3} \rightarrow \frac{6x^4}{12} \\
 &\quad \quad \quad \rightarrow \frac{6x^5}{60} \rightarrow \frac{6x^7}{420}
 \end{aligned} \right] \\
 \end{aligned}$$

$$PI = e^{x+y} \left[\frac{x^4 y^2}{12} + \frac{x^5 y}{15} + \frac{x^7}{70} \right]$$