



TYPE 3

$$f(x, p, q) = 0 \quad p = \frac{\partial z}{\partial x} \quad q = \frac{\partial z}{\partial y}$$
$$f(x, p, q) = 0 \quad p(1+q) = qx$$

J. solve  $p(1+q) = qx$

Soln.

$$\text{Given: } p(1+q) = qx \quad p + q = \frac{qx}{p}$$

$$\text{Let } u = x + ay$$

$$\text{Then: } p = \frac{\partial z}{\partial x} = \frac{dx}{du}$$



# SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore-641035.



## UNIT-III PARTIAL DIFFERENTIAL EQUATIONS

## Solution of First Order Partial Differential Equations

TYPE - III

$$f(x, p, q) = 0$$

Ex. Solve  $p(1+q) = qx \rightarrow (1)$

Soln.:

Let  $u = x + ay$

Then  $p = \frac{dx}{du}$  and  $q = a \frac{dx}{du}$

$$(1) \Rightarrow \frac{dx}{du} \left( 1 + a \frac{dx}{du} \right) = a \frac{dx}{du} x$$

$$1 + a \frac{dx}{du} = ax$$

$$a \frac{dx}{du} = ax - 1$$

$$\frac{dx}{du} = \frac{ax-1}{a}$$

$$\frac{du}{dx} = \frac{a}{ax-1}$$

$$du = \frac{a}{ax-1} dx$$

Integrating,

$$u = \int \frac{a}{ax-1} dx$$

$$u = \log(ax-1) + \log c$$

$$x+ay = \log [c(ax-1)]$$

Ex. Solve  $x^2 = 1 + p^2 + q^2$

Soln.

$$x^2 = 1 + p^2 + q^2 \rightarrow (1)$$

Let  $u = x + ay$

$$p = \frac{dx}{du} \Rightarrow q = a \frac{dx}{du}$$



**SNS COLLEGE OF TECHNOLOGY**  
 (An Autonomous Institution)  
 Coimbatore-641035.



UNIT-III PARTIAL DIFFERENTIAL EQUATIONS

Solution of First Order Partial Differential Equations

$$\begin{aligned}
 \text{(1) } z^2 &= 1 + \left(\frac{dz}{du}\right)^2 + \left(\frac{dz}{dv}\right)^2 \\
 z^2 &= \left(\frac{dz}{du}\right)^2 (1 + \alpha^2) + 1 \\
 z^2 - 1 &= \left(\frac{dz}{du}\right)^2 (1 + \alpha^2) \\
 \left(\frac{dz}{du}\right)^2 &= \frac{z^2 - 1}{1 + \alpha^2} \\
 \frac{dz}{du} &= \sqrt{\frac{z^2 - 1}{1 + \alpha^2}} = \frac{\sqrt{z^2 - 1}}{\sqrt{1 + \alpha^2}} \\
 \frac{dz}{\sqrt{z^2 - 1}} &= \frac{du}{\sqrt{1 + \alpha^2}}
 \end{aligned}$$

Integrating on both sides,

$$\begin{aligned}
 \cosh^{-1} z &= \frac{1}{\sqrt{1 + \alpha^2}} u + C \\
 &= \frac{1}{\sqrt{1 + \alpha^2}} (x + \alpha y) + C
 \end{aligned}$$

Hence solve  $p(1 + q^2) = q(r - a)$

$$\begin{aligned}
 x &= \bar{P}v - a & r &= \bar{x} - \bar{P}v \\
 x - a &= \bar{P}v & x + a &= \bar{q}v
 \end{aligned}$$