(23). $\int_{-1}^{1} (x^4 - 3x + 7)\, dx$

Sol:

$2n - 1 = 4$

$2n = 5$

$n = 2.5 \simeq 3.$

$f(x) = x^4 - 3x + 7.$

$x_1 = \sqrt{3/5} = 0.774596669$

$x_2 = 0$

$x_3 = -\sqrt{3/5} = 0.774596669$

$w_1 = 5/9 = 0.5555$

$w_2 = 8/9 = 0.8888$

$w_3 = 5/9 = 0.5555$

WKT, $f(x_1) = x_1^4 - 3x_1 + 7$

$= (0.774596669)^4 - 3(0.774596669) + 7$

$f(x_1) = 5.0362$

$w_1 f(x_1) = 0.555 \times 5.0362 = 2.798$

$f(x_2) = (0)^4 - 3(0) + 7 = 7$

$w_2\, f(x_2) = 7 \times 0.888 = 6.216$

$w_3\, f(x_3) = 8.96\ \times 0.555 = 4.977$

$\therefore w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3) = 2.798 + 6.216 + 4.977$

$= 13.996.$

14.397

$$\int_{-1}^{1} (x^4 - 3x + 7)\, dx = \left(\frac{x^5}{5} - 3\frac{x^2}{2} + 7x \right)_{-1}^{1}$$

$$= \left(\frac{1}{5} - \frac{3}{2} + 7\right) - \left(-\frac{1}{5} - \frac{3}{2} - 7\right)$$

$$= \left(\frac{2 - 15 + 70}{10}\right) - \left(\frac{-2 - 15 - 70}{10}\right)$$

$$= 10 - (-3.996)$$

$$= 13.996$$

$14.44.$

(3) Evaluate $\int_{-1}^{1} e^{-x}\, dx$ by applying 3 point Gausion quadrature.

$f(x) = e^{-x}$

$x_1 = \sqrt{3/5} = 0.774596669$

$x_2 = 0$

$x_3 = -\sqrt{3/5} = 0.774596669$

$w_1 = 5/9 = 0.5555, \quad w_2 = 8/9 = 0.8888$

$w_3 = 0.5555$

WKT,

$f(x) = e^{-x}$

$f(x_1) = e^{-x_1} = e^{-0.774596669}$

$w_1 f(x_1) = 0.460889639 \times 0.5555$
$= 0.256024191$

$f(x_2) = e^{-x_2} = e^{-0} = 1 \times 0.8888 = 0.8888$ $f(x_2) \times w_2$

$f(x_3) = e^{-x_3} = e^{0.774596669}$

$= 2.169716837$

$w_3 f(x_3) = 2.169716237 \times 0.5555$

$= 1.2052777$

$w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3) =$

$0.256024191 + 0.8888 + 1.2052777$

$= \underline{2.350}$

(4) Evaluate $I = \int_{-1}^{1} \cos \pi x/2 \, dx$ by applying 3 point gaussion quadrature ~~and~~

$f(x) = \cos \pi x / 2$

$x_1 = 0.774596669 = \sqrt{3/5}$

$x_2 = 0$

$x_3 = -0.774596669 = -\sqrt{3/5}$

$w_1 = 5/9 = 0.5555$

$w_2 = 8/9 = 0.8888$

$w_3 = 5/9 = 0.5555$

$f(x_1) = \cos \frac{\pi x_1}{2}$

$= 0.3467$

$w_1 f(x_1) = 0.3467 \times 0.5555 = 0.1926$

$f(x_2) = \cos \frac{\pi(0)}{2} = 1$

$w_2 f(x_2) = 1 \times 0.888 = 0.888$

$f(x_3) = \cos \frac{\pi (0.774596669)}{2}$

$= 0.3467$

$w_3 f(x_3) = 0.3467 \times 0.5555 = 0.1926$

$w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3)$

$= 0.1926 + 0.8888 + 0.1926$

$= 1.2741$

$\int_{-1}^{1} \cos \pi x/2 \, dx = \left[\frac{\sin \pi x/2}{\pi/2} \right]_{-1}^{1}$

$= 2/\pi \left[\sin \pi/2 - \sin -\pi/2 \right]$

$[1-(-1)]$

$= 4/\pi = 1.2732395$

(5) Integrate the function $f(x) = 1 + x + x^2 + x^3$ between the limits (-1 to 1) using (1) exact method and (2) Gauss integration method and compare the two results.

Sol:

$2n - 1 = 3$

$2n = 4$

$\boxed{n = 2}$

2 point values

$x_1 = \sqrt{1/3} = 0.577350269$

$x_2 = -\sqrt{1/3} = -0.577350269$

$w_1 = 1$

$w_2 = 1$

$f(x_1) = 1 + 0.577350269 + (0.577350269)^2 + (0.577350269)^3$

$= 2.103133692$

$w_1 f(x_1) = 2.103133692$

$f(x_2) = 1 - 0.577350269 + (-0.577350269)^2 + (-0.577350269)^3$

$= 0.563532974$

$w_2 f(x_2) = 0.56353297 4$

$$w_1 f(r_1) + w_2 f(r_2) = 2.6666$$

$$\int_{-1}^{1} (1 + r + r^2 + r^3)\, dr = \left[r + \frac{r^2}{2} + \frac{r^3}{3} + \frac{r^4}{4} \right]_{-1}^{1}$$

$$= \left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \right] - \left[-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} \right]$$

$$= 2.0833 - (-0.5833)$$

$$= 2.6666$$