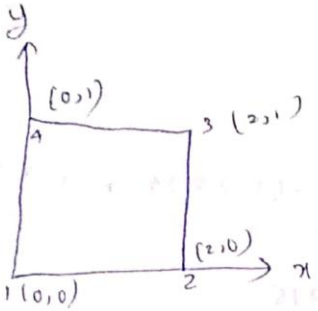
(3) A 4 noded rectangular element as shown. Determine the following (i) Jacobian matrix (2) strian dispracement matrix (3) Element stress matrix.



Take $E = 2 \times 10^5$ N/mm^2

Poisson ratio $V = 0.25$

$$u = [0, 0, 0.003, 0.004, 0.006, 0.004, 0, 0]^T$$

$s = 0, \eta = 0.$

$$u = \begin{bmatrix} 0 \\ 0 \\ 0.003 \\ 0.004 \\ 0.006 \\ 0.004 \\ 0 \\ 0 \end{bmatrix}$$

Sol:

$x_1 = 0, y_1 = 0$

$x_2 = 2, y_2 = 0$

$x_3 = 2, y_3 = 1$

$x_4 = 0, y_4 = 1.$

$$[J] = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

$$J_{11} = \frac{1}{4}\left[-(1-\eta)x_1 + (1-\eta)x_2 + (1+\eta)x_3 - (1+\eta)x_4\right]$$

$J_{11} = 1$

$$J_{12} = 0$$

$$J_{21} = 0$$

$$J_{22} = 0.5$$

$$|J| = \begin{vmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 \\ 0 & 0.5 \end{vmatrix} = 0.5$$

$$[B] = \frac{1}{|J|} \begin{bmatrix} J_{22} & J_{12} & 0 & 0 \\ 0 & 0 & -J_{21} & J_{11} \\ -J_{21} & J_{11} & J_{22} & -J_{12} \end{bmatrix} \times \frac{1}{4} \begin{bmatrix} -(1-\eta) & 0 & (1-\eta) & 0 & (1+\eta) & 0 & -(1+\eta) & 0 \\ -(1-\xi) & 0 & -(1+\xi) & 0 & 1+\xi & 0 & (1-\xi) & 0 \\ 0 & -(1-\eta) & 0 & (1-\eta) & 0 & (1+\eta) & 0 & -(1+\eta) \\ 0 & -(1-\xi) & 0 & -(1+\xi) & 0 & (1+\xi) & 0 & (1-\xi) \end{bmatrix}$$

$$= \frac{1}{0.5} \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0.5 & 0 \end{bmatrix} \times \frac{1}{4} \begin{bmatrix} -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ -1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & -1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$$\frac{1}{2} = 0.25 \begin{bmatrix} -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -2 & 0 & -2 & 0 & 2 & 0 & 2 \\ -2 & -1 & -2 & +1 & 2 & 2 & 1 & -1 \end{bmatrix}$$

$$[D] = \frac{2\times 10^5}{1-0.25^2}\begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & \frac{0.75}{2} \end{bmatrix}$$

$$[D] = 53333.33\begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

$$[D][B] = 53333.33\begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix} \times 0.25 \begin{bmatrix} -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -2 & 0 & -2 & 0 & 2 & 0 & 2 \\ -2 & -1 & -2 & 1 & 2 & 2 & 1 & -1 \end{bmatrix}$$

$$= 13,333.33 \begin{bmatrix} -4 & -2 & 4 & -2 & 4 & 2 & -4 & 2 \\ -1 & -8 & 0 & -8 & 1 & 8 & -1 & 8 \\ -3 & -1.5 & -3 & 1.5 & 3 & 3 & 1.5 & -1.5 \end{bmatrix}$$

$$[\sigma] = [D][B]\{u\}$$

$$= 13,333.33 \begin{bmatrix} -4 & -2 & 4 & -2 & 4 & 2 & -4 & 2 \\ -1 & -8 & 0 & -8 & 1 & 8 & -1 & 8 \\ -3 & -1.5 & -3 & 1.5 & 3 & 3 & 1.5 & -1.5 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0.003 \\ 0.004 \\ 0.006 \\ 0.004 \\ 0 \\ 0 \end{Bmatrix}$$

$$= 13.333.33$$