ISOPARAMETRIC ELEMENTS

Gaussian Quadrature and Applications to Plane Stress Problem

no of Points n	location x_i	corresponding weights (W_i)
1	$x_1 = 0.000$	2.000
2	$x_1, x_2 = \pm\sqrt{1/3}$ $= \pm 0.577350269$	1.000
3	$x_1, x_3 = \pm\sqrt{3/5}$, $x_2 = 0.00$ $= \pm 0.774596669$	$5/9 = 0.555555$ $x_2 = 8/9 = 0.8888$
4	$x_1, x_4 = \pm 0.8611363116$ $x_2, x_3 = \pm 0.3399810436$	~~$x_2 = x_3 = 0.8888$~~ 0.3478548451 0.6521451549

Evaluate $\int_{-1}^{1} (x^4 + x^2)\,dx$ by applying 3point gaussian quadrature.

Given data :

integral $I = \int_{-1}^{1} (x^4 + x^2)\,dx$

$f(x) = x^4 + x^2$

$x_1 = \sqrt{3/5} = 0.774596669$

$x_2 = 0$

$x_3 = -\sqrt{3/5} = -0.774596669$

weight

$w_1 = 5/9 = 0.55555$

$w_2 = 8/9 = 0.88888$

$w_3 = 5/9 = 0.55555$

WKT,

$f(x) = x^4 + x^2$

$f(x_1) = x_1^4 + x_1^2$

$= (0.774596669)^4 + (0.774596669)$

$f(x_1) = 0.959999$

$w_1 f(x_1) = 0.555 \times 0.96$

$= 0.53328$

$f(x_2) = x_2^4 + x_2^2$

$f(x_2) = 0$

$w_2 f(x_2) = 0$

$f(x_3) = (-0.774596669)^4 + (-0.774596669)^2$

$= 0.96$

$w_3 f(x_3) = 0.96 \times 0.555$

$= 0.53328$

$(w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3)$

$= 0.53328 + 0 + 0.53328$

$= 1.0666$