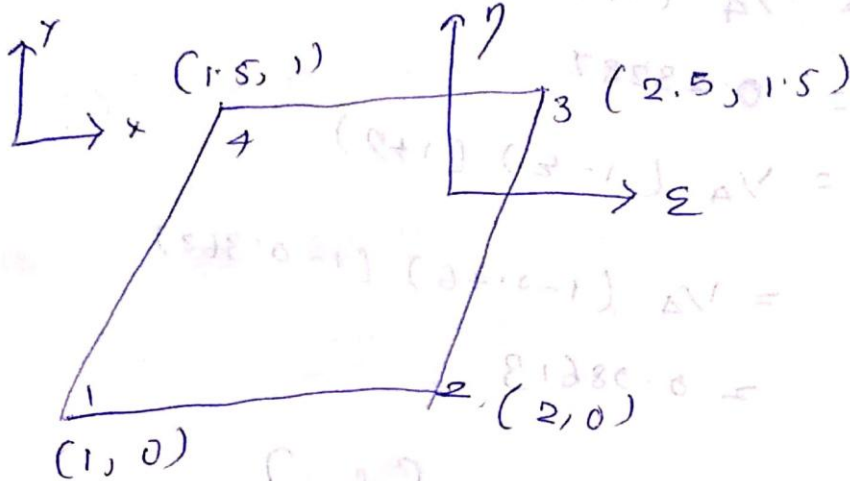




Q Evaluate isoparametric

the Jacobian matrix quadrilateral element

for the as shown.



SOL:

$$x_1 = 1, \quad y_1 = 0$$

$$x_2 = 2, \quad y_2 = 0$$

$$x_3 = 2.5, \quad y_3 = 1.5$$

$$x_4 = 1.5, \quad y_4 = 1$$

Shape function

$$N_1 = \frac{1}{4} [(1-\xi)(1-\eta)]$$

$$N_2 = \frac{1}{4} [(1+\xi)(1-\eta)]$$

$$N_3 = \frac{1}{4} [(1+\xi)(1+\eta)]$$

$$N_4 = \frac{1}{4} [(1-\xi)(1+\eta)]$$



$$\begin{aligned}
 x &= w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4 \\
 &= \frac{1}{4} \left[1(1-\varepsilon)(1-\eta) + 2(1+\varepsilon)(1-\eta) + \right. \\
 &\quad \left. 2.5(1+\varepsilon)(1+\eta) + 1.5(1-\varepsilon)(1+\eta) \right] \\
 x &= \frac{1}{4} [7 + \eta - 2\varepsilon + 0\varepsilon\eta] \\
 y &= \frac{1}{4} \left[0 + 0 + 1.5(1+\varepsilon)(1+\eta) + 1(1-\varepsilon)(1+\eta) \right] \\
 &= \frac{1}{4} [1.5 + 1.5\eta + 1.5\varepsilon + 1.5\varepsilon\eta + 1 + \eta - \varepsilon - \varepsilon\eta] \\
 &= \frac{1}{4} [2.5 + 2.5\eta + 0.5\varepsilon + 0.5\varepsilon\eta] \\
 [J]_2 &= \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \\
 J_{11} &= \frac{\partial x}{\partial \varepsilon} = \frac{1}{4} [0 + 0 + 2] = \frac{1}{2} = 0.5 \\
 J_{12} &= \frac{\partial x}{\partial \eta} = \frac{1}{4} [0 + 0 + 0.5 + 0.5\varepsilon] = \frac{1}{8} [1 + \varepsilon] \\
 J_{21} &= \frac{\partial y}{\partial \varepsilon} = \frac{1}{4} [0] = \frac{1}{4} \\
 J_{22} &= \frac{\partial y}{\partial \eta} = \frac{1}{4} [2.5 + 0.5\varepsilon] \\
 &= 0.125 [5 + \varepsilon] \\
 [J] &= \begin{bmatrix} 0.5 & 0.5/4 [1 + \eta] \\ 1/4 & 0.125 [5 + \varepsilon] \end{bmatrix}
 \end{aligned}$$