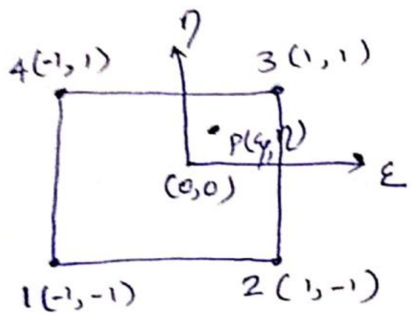
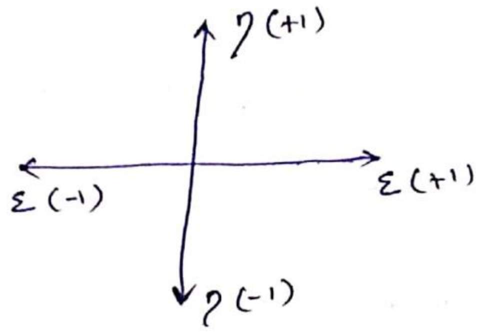
UNIT-5

Isoparametric Formulation

Shape Functions For 4 Noded Rectangular Parent element by using Natural co-ordinate system and co-ordinate Transformation.





At node 1: $\xi = -1, \ \eta = -1$

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta)$$

$$N_2 = \frac{1}{4}(1+\xi)(1-\eta)$$

$$N_3 = \frac{1}{4}(1+\xi)(1+\eta)$$

$$N_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

The displacement components,

$$u = N_1 u_1 + N_2 u_2 + N_3 u_3 + N_4 u_4$$

and

$$v = N_1 v_1 + N_2 v_2 + N_3 v_3 + N_4 v_4$$

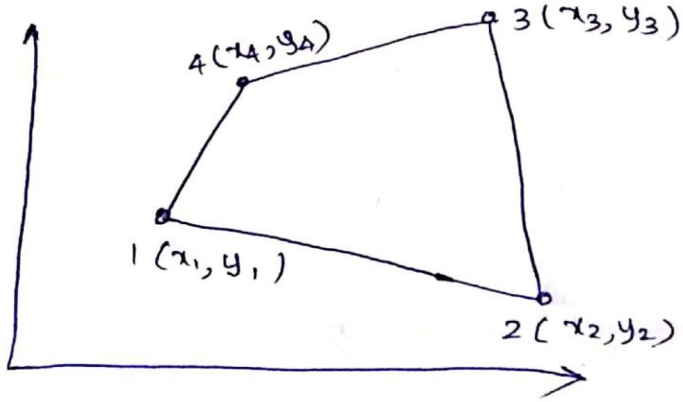
It can be written in matrix form as,

$$u = \begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}$$

In the isoparametric formulation i.e, for global system, the co-ordinates of the nodal points are (x_1, y_1), (x_2, y_2), (x_3, y_3) and (x_4, y_4). In order to get mapping,

$$x = N_1 x_1 + N_2 x_2 + N_3 x_3 + N_4 x_4$$

$$y = N_1 y_1 + N_2 y_2 + N_3 y_3 + N_4 y_4$$



The above equations can be written in matrix form,

$$u = \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix} \begin{Bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ x_3 \\ y_3 \\ x_4 \\ y_4 \end{Bmatrix}$$