Finite Element Equations for One-Dimensional Heat conduction Problems:-

w.k.t,

$$\{F\} = [K_c]\{T\}$$

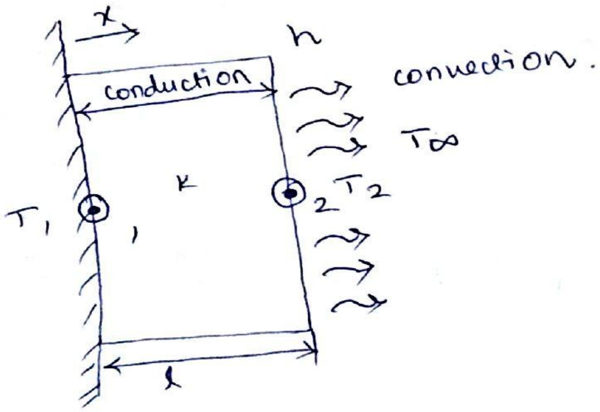
$\{T\}$ → Nodal Temperature.

$\{F\}$ → Element force vector

$[K_c]$ → Stiffness matrix

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{AK}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix}$$

Case (i) one dimensional heat conduction with free end convection.



Consider a one dimensional element with nodes 1 and 2. T_1 and T_2 are the temperatures at the respective nodes. Assume convection occurs only from the right end of the element.

Stiffness matrix $[K_c]$ for one dimensional heat conduction element is given by

$$[K_c] = \frac{AK}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

The convection term contribution to the stiffness matrix is

$$[K_h]_{end} = \iint_A h\,[N]^T [N]\, dA$$

where, h = Heat transfer co-efficient, $W/m^2 K$

N = shape factor,

WKT, shape factor, $[N] = [N_1 \quad N_2] = \left[\frac{l-x}{l} \quad \frac{x}{l}\right]$

At node 2, $x = l$

$$[N] = [N_1 \quad N_2] = [0 \quad 1]$$

$$[N]^T = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

Substitute $[N]$ and $[N]^T$ values in equ,

$$[K_h]_{end} = \iint_A h \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} [0 \quad 1]\, dA$$

$$[K_h]_{end} = h \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \int dA$$

$$= hA \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Stiffness matrix $[K] = [K_c] + [K_h]$

$$[K] = \frac{AK}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + hA \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

The convection force from the free end of the element is obtained from the following,

$$\{F_h\}_{end} = h T_\infty A \begin{Bmatrix} N_1 & (x=l) \\ N_2 & (x=l) \end{Bmatrix}$$

$$\{F_h\}_{end} = h T_\infty A \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

General force equation is,

$$\{F\} = [K]\{T\}$$

Substitute $\{F\}$ $[K]$ values,

$$h T_\infty A \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} = \left[\frac{AK}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + hA \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right] \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix}$$

where,

A = Area of the element, m^2

K = Thermal conductivity of the element, W/mK

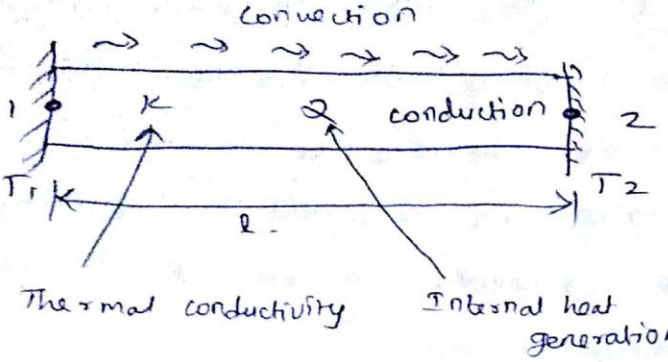
l = Length of the element,

h = Heat transfer co-efficient, W/m^2K

T_∞ = Fluid Temperature, K

T = Temperature, K.

Case (ii) one dimensional element with conduction, convection and internal heat generation.



Consider a rod with nodes 1 and 2. This rod subjected to conduction, convection and Internal heat generation.

$$\left[\frac{AK}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{hPl}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \right] \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix}$$

$$= \frac{QAl + PhT_\infty l}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

where,

- $A \rightarrow$ Area of the element, m^2
- $K \rightarrow$ Thermal conductivity of the element, W/mK
- $l \rightarrow$ Length of the element, m
- $h \rightarrow$ Heat transfer co-efficient, $W/m^2 K$
- $P \rightarrow$ Perimeter, m
- $T \rightarrow$ Temperature, K
- $Q \rightarrow$ Heat generation, W
- $T_\infty \rightarrow$ Fluid Temperature, K.