Problem ①

A wall of 0.6 m thickness having thermal conductivity of 1.2 W/mK. The wall is to be insulated with a material of thickness 0.06 m having an average thermal conductivity of 0.3 W/mK. The inner surface temperature is 1000°C and outside of the insulation is exposed to atmospheric air at 30°C with heat transfer co-efficient of 35 W/m²K. Calculate the nodal temperature.

Thickness of the wall, $l_1 = 0.6$ m.

Thermal conductivity of the wall, $k_1 = 1.2$ W/mK

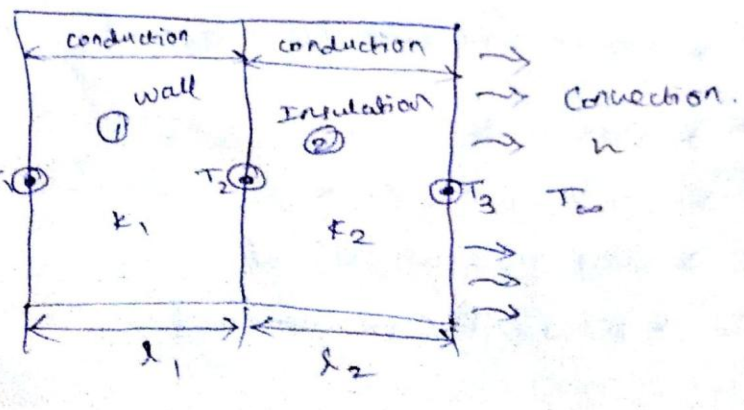
Thickness of the insulation, $l_2 = 0.06$ m,

$k_2 = 0.3$ W/mK.

Inner Surface Temperature, $T_1 = 1000°C + 273 = 1273$ K.

Atmospheric air temperature, $T_\infty = 30°C + 273 = 303$ K.

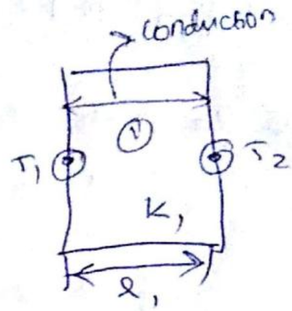
heat transfer co-efficient, $h = 35$ W/m² K



To Find: Nodal temperatures, (T_2 & T_3) ⑥

Solution:

For element 1: (Nodes 1, 2)



Finite element equation is,

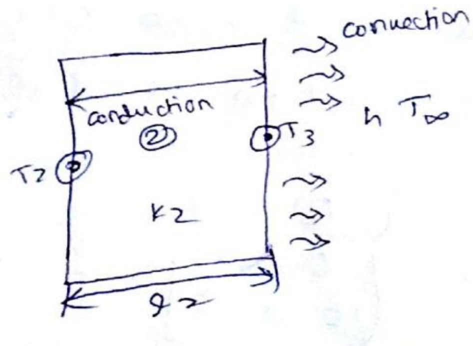
$$\frac{A_1 k_1}{l_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}T_1\\ T_2\end{Bmatrix} = \begin{Bmatrix}F_1\\ F_2\end{Bmatrix}$$

For unit area $A_1 = 1\ m^2$

$$\frac{1.2}{0.6}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}T_1\\ T_2\end{Bmatrix} = \begin{Bmatrix}F_1\\ F_2\end{Bmatrix}$$

$$\begin{bmatrix}2 & -2\\ -2 & 1\end{bmatrix}\begin{Bmatrix}T_1\\ T_2\end{Bmatrix} = \begin{Bmatrix}F_1\\ F_2\end{Bmatrix}$$

For element 2: (Nodes 2, 3)



This element is subjected to both conduction and convection.

So, the finite element equation is,

$$\left(\frac{A_2 k_2}{l_2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + hA\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\right)\begin{Bmatrix}T_2\\ T_3\end{Bmatrix} = hT_\infty A\begin{Bmatrix}0\\ 1\end{Bmatrix}$$

$$\left(\frac{1\times 0.3}{0.06}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + 35\times 1\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\right)\begin{Bmatrix}T_2\\ T_3\end{Bmatrix} = 35\times 303\times 1\begin{Bmatrix}0\\ 1\end{Bmatrix}$$

$$\begin{bmatrix} 5 & -5 \\ -5 & 40 \end{bmatrix} \begin{Bmatrix} T_2 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 10.605\times10^3 \end{Bmatrix}$$

Assemble the finite element equations,

$$\begin{bmatrix} 2 & -2 & 0 \\ -2 & 2+5 & -5 \\ 0 & -5 & 40 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

In this problem there is no heat generation,

$\{F_1\} = \{F_2\} = 0$, $\{F_3\} = 10.605\times10^3$

$$\begin{bmatrix} 2 & -2 & 0 \\ -2 & 7 & -5 \\ 0 & -5 & 40 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 10.605\times10^3 \end{Bmatrix}$$

Step 1: The first row and first column of the stiffness matrix [K] have been set equal to 0 except for the main diagonal,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 7 & -5 \\ 0 & -5 & 40 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 10.605\times10^3 \end{Bmatrix}$$

Step 2: The first row of the force matrix is set (or) replaced by known temperature at node 2

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 7 & -5 \\ 0 & -5 & 40 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} 1273 \\ 0 \\ 10.605\times10^3 \end{Bmatrix}$$

Step 3 : The second row, first column of stiffness matrix [K] value is multiplied by known temperature at node 1, (i.e. $-2 \times 1273 = -2546$)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 7 & -5 \\ 0 & -5 & 40 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} 1273 \\ 2546 \\ 10.605 \times 10^3 \end{Bmatrix}$$

Solving the above equations,

$$7T_2 - 5T_3 = 2546$$

$$-5T_2 + 40T_3 = 10.605 \times 10^3$$

$$\boxed{T_2 = 607.313\ K}$$

$$\boxed{T_3 = 341.03\ K}$$

Nodal Temperatures:

$T_1 = 1273\ K$,

$T_2 = 607.313\ K$

$T_3 = 341.03\ K$