CONDUCTION

PROBLEMS & SOLUTIONS

Problem

The inner surface of furnace wall is at 200^0C and outer surface at 50^0C. Calculate the heat lost per m^2 area of the wall. If thermal conductivity of the brick is $0.5\ W/m^0C$ & the wall thickness is 200mm.

Sol: Given $T_1 = 200^0C$

$T_2 = 50^0C$

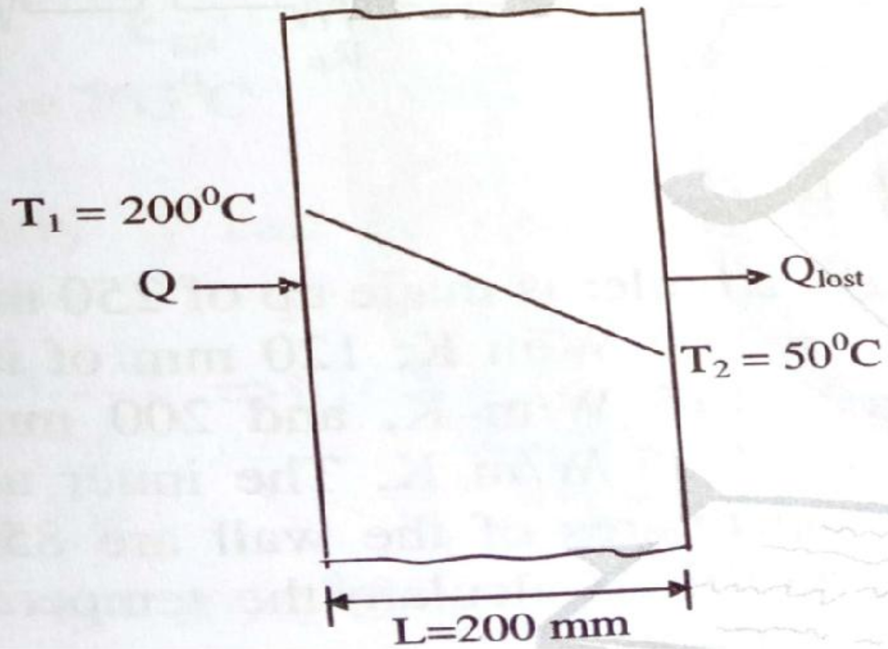
$L = 200 \text{ mm} = 0.2$

$k = 0.5 \text{ W/m}^0\text{C}$

$$\text{Heat lost} = -\,kA\,\frac{dT}{dx}$$

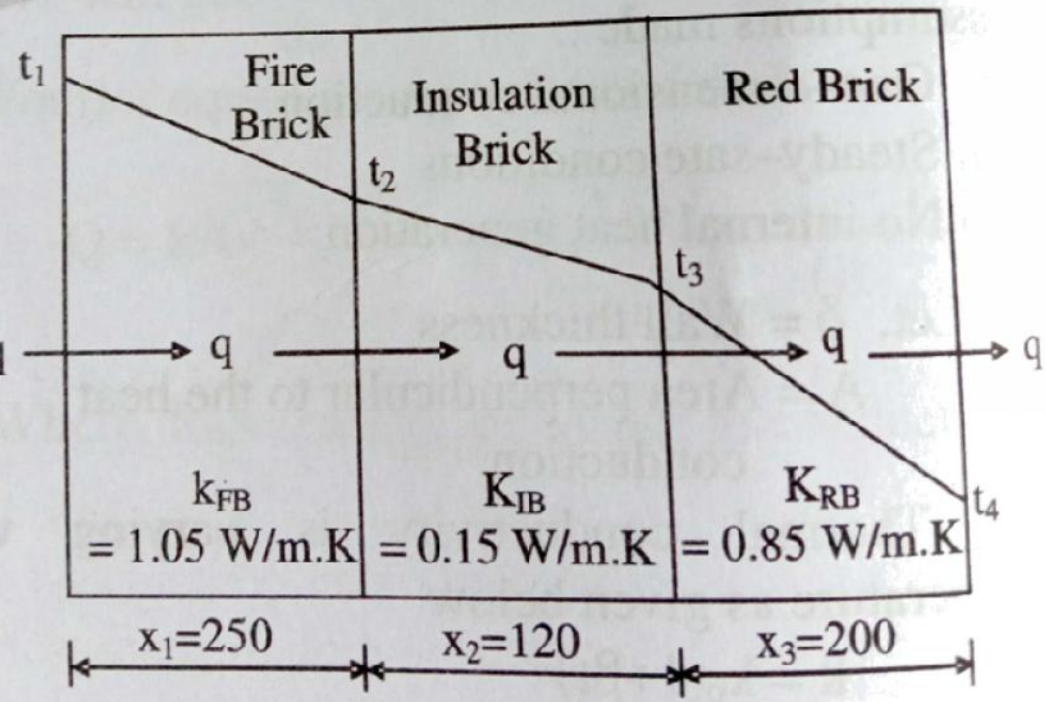
$$\text{Heat lost/m}^2 = \frac{-K(T_2 - T_1)}{L}$$

$$= \frac{-0.5(50-200)}{0.2} = 375 \text{ W/m}^2$$



The wall of a boiler is made up of 250mm of the brick, K_{FB} = 1.05 W/m K; 120 mm of insulation brick K_{IB} = 0.15 W/m K, and 200 mm of red brick, K_{RB}= 0.85 W/m K. The inner and outer surface temperatures of the wall are 850^0C and 65^0C respectively. Calculate the temperatures at the contact surfaces.

Sol:



t1
Fire Brick
Insulation Brick
Red Brick
t2
t3
q
q
q
q
q
kFB = 1.05 W/m.K
KIB = 0.15 W/m.K
KRB = 0.85 W/m.K
t4
x1=250
x2=120
x3=200

The rate of heat flow per square meter is

$$\frac{Q}{A} = q = \frac{(t_1 - t_4)}{\frac{x_1}{k_{FB}} + \frac{x_2}{k_{IB}} + \frac{x_3}{k_{RB}}}$$

$$\frac{Q}{A} = \frac{1 \times (850 - 65)}{\frac{0.250}{1.05} + \frac{0.120}{0.15} + \frac{0.200}{0.85}}$$

$$= 616.5 \text{ W/m}^2$$

$$\frac{Q}{A} = \frac{t_1 - t_2}{\frac{x_1}{k_{FB}}} = \frac{t_2 - t_3}{\frac{x_2}{k_{IB}}} = \frac{t_3 - t_4}{\frac{x_3}{k_{RB}}}$$

$$\therefore \quad \frac{Q}{A} = \frac{t_1 - t_2}{\frac{x_1}{k_{FB}}} \Rightarrow 616.5 = \frac{850 - t_2}{\frac{0.250}{1.05}}$$

$$\Rightarrow \quad t_2 = 703^0\text{C}$$

Similarly t_3 can be determined from the relation

$$\frac{Q}{A} = \frac{t_2 - t_3}{\frac{x_2}{k_{IB}}} \Rightarrow 616.5 = \frac{703 - t_3}{\frac{0.120}{0.15}}$$

$$\therefore \Rightarrow \quad t_3 = 209.8^0\text{C}$$

The Boiler wall is made up of two layers, A & B. Thickness & Thermal conductivity of A are 240 mm and 0.2 W/m^{0}C respectively. For B, thickness & thermal conductivity are 525 mm and 0.3 W/m^{0}C respectively. Inner surface of A is maintained at 1000^{0}C and outer surface of B is maintained at 250^{0}C. If there is contact thermal resistance of 0.05^{0}C/W per unit area exists at the interface.

Cal. i. the heat lost per m^2 area

II. The temperature drop at the interface

Sol: Given, $T_1 = 1000^0C$, $T_4 = 250^0C$

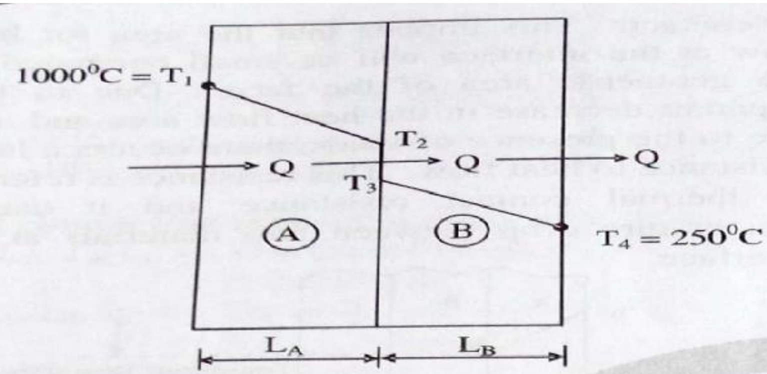
$L_A = 240\text{ mm} = 0.24\text{ m}$

$L_B = 525\text{ mm} = 0.525\text{ m}$

$K_A = 0.2\text{ W/m}^0C$

$K_B = 0.3\text{ W/m}^0C$

$(R_{th})_{contact} = 0.05^0C/W$



Heat Lost per m² area.

$$\frac{Q}{A} = q = \frac{\Delta T}{\Sigma R_{th}} = \frac{\Delta T}{R_{th-A} + R_{th-contact} + R_{th-B}}$$

$$= \frac{T_1 - T_4}{\frac{L_A}{K_A} + R_{th-contact} + \frac{L_B}{K_B}}$$

$$= \frac{1000 - 250}{\frac{0.24}{0.2} + 0.05 + \frac{0.525}{0.3}}$$

$$= \frac{750}{3} = 250\, W/m^2$$

And also we can write.

$$q = \frac{T_1 - T_2}{\frac{L_a}{K_A}} = \frac{T_3 - T_4}{\frac{L_B}{K_B}}$$

$$250 = \frac{1000 - T_2}{\frac{0.240}{0.2}} = \frac{T_3 - 250}{\frac{0.525}{0.3}}$$

$$1000 - T_2 = 250 \times \frac{0.24}{0.2}$$

$$1000 - T_2 = 300$$

$$\Rightarrow T_2 = 700^0 C$$

$$T_3 - 250 = 250 \times \frac{0.525}{0.3}$$

$$T_3 = 687.5$$

$\therefore$ Temperature drop at interface $= T_2 - T_3$

$= 700 - 687.5$

$= 12.5^0C$

A furnace wall 200 mm thick is made of a material having thermal conductivity of 1.45 W/m. K. The inner and outer surface are exposed to average temperatures of 350^0C and 40^0C respectively. If the gas and air film coefficients are 58 and 11.63 W/m^2 K respectively, find the rate of heat transfer through a wall of 2.5 square meters. Also, find the temperatures on the two sides of the wall.

Sol: Given, $x = 200\text{ mm} = 0.2\text{ m}$,
$k = 1.45\text{ W/mK}, A = 2.5\text{ m}^2$
$T_A = 350^0C = 623\text{ K}, T_B = 40^0C = 313\text{ K}$,
$h_A = 58\text{ W/m}^2\text{ K}, \; h_B = 11.63\text{ W/m}^2\text{K}$,

Rate of heat transfer, $Q = \dfrac{A(T_A - T_B)}{\dfrac{1}{h_A} + \dfrac{x}{k} + \dfrac{1}{h_B}}$

$$= \frac{2.5(623-313)}{\frac{1}{58} + \frac{0.2}{1.45} + \frac{1}{11.63}} = 3214\text{ J/s}$$

Let, T_1 = Temperature on the inner side of the wall, and

T_2 = Temperature on the outside of the wall.

$$Q = h_A.A(T_A - T_1)$$
$$3214 = 58 \times 2.5(623 - T_1)$$
$$\Rightarrow T_1 = 600.84\text{ K} = 327.84^0\text{C}$$

Similarly

$$Q = h_B A(T_2 - T_B)$$
$$3214 = 11.63 \times 2.5(T_2 - 313)$$
$$T_2 = 423.5\text{ K} = 150.5^0\text{C}$$

A steam pipe of inner diameter 200 mm is covered with 50mm thick high insulated material of thermal conductivity k = 0.01 W/m^0C. The inner and outer surface temperatures maintained at 500^0C and 100^0C respectively. Calculate the total heat loss per meter length of pipe?

Sol: Given, $r_1 = \frac{200}{2} = 100\,mm = 0.1\,m$

$$r_2 = \frac{200+100}{2} = 150\,mm = 0.15\,m$$

$$k = 0.01\ W/m^0C$$

$$T_1 = 500^0C,\ T_2 = 100^0C$$

Heat transfer is given by

$$Q = \frac{T_1 - T_2}{\frac{\ell n\left(\frac{r_2}{r_1}\right)}{2\pi k \ell}} = \frac{500-100}{\frac{\ell n\left(\frac{0.15}{0.1}\right)}{2\pi \times 0.01 \times 1}}$$

$$= 61.98\ W.$$

A 25 cm steam main 225 meter long is covered with 5 cm of high temperature insulation (k=0.095 W/m K) and 4 cm of low temperature insulation (k=0.065 W/m K). The inner and outer surface temperatures as measured are 400^0C and 50^0C respectively. Neglect heat conduction through pipe material.

Determine

i. The total heat loss per hour.

ii. The total heat loss per sq. m of outer surface.

iii. The heat loss per sq.m of pipe surface.

iv. The temperature between the two layers of insulation.

Sol: Outside diameter of pipe, $d_1 = 25$ cm

Outside diameter of first layer,

$$d_2 = 25 + 2 \times 5 = 35 \text{ cm}$$

Outside diameter of second layer,

$$d_3 = 35 + 2 \times 4 = 43 \text{ cm}$$

$k_1 = 0.095$ W/m K, $k_2 = 0.065$ W/m K

(i) Total heat loss,

$$Q_{total} = \frac{2\pi\ell(T_1 - T_3)}{\frac{ln(d_2/d_1)}{k_1} + \frac{ln(d_3/d_2)}{k_2}}$$

$$= \frac{2\pi \times 225(400 - 50)}{\frac{ln(35/25)}{0.095} + \frac{ln(43/35)}{0.065}}$$

$$= 73754 \text{ W} = \frac{73754 \times 3600}{1000} \text{kJ/h}$$

$$= 265515 \text{ kJ/h}$$

(ii) Total heat lost per sq. m of outer surface

$$= \frac{Q_{total}}{\pi d_3 \ell} = \frac{265515}{\pi \times 0.43 \times 225}$$

$$= 873.5 \text{kJ/h}$$

(iii) Heat loss per sq. m of pipe surface

$$= \frac{Q_{\text{total}}}{\pi d_1 \ell} = \frac{265514}{\pi \times 0.25 \times 225}$$

$$= 15025.5 \text{kJ} / \text{h}$$

(iv) For temperature between two layers

$$\frac{Q}{\ell} = \frac{2\pi k(t_1 - t_2)}{ln(d_2 / d_1)}$$

$$\frac{73754}{225} = \frac{2\pi \times 0.095(400 - t_2)}{ln(35/25)}$$

$$\therefore \quad t_2 = 215^0 \text{C}$$

Water is pumped through an iron pipe (k=67.2 W/m^2 K), 2 meters long as the rate of 1000kg/min. The inner and outer diameters of the tube are 50mm and 60mm respectively. Calculate the rise in temperature of water when the outside of the tube is heated to a temperature of 600^{0}C. The initial temperature of the water is 30^{0}C.

Sol: Given, $k = 67.2$ W/mK, $L = 2$m,

$m = 1000$ kg/min

$r_1 = 25$ mm $= 0.025$ m,

$r_2 = 30$ mm $= 0.03$ m,

$T_1 = 600^0C = 873$ K,

$T_{wl} = 30^0C = 303$ K

Let T_{w2} = Final temperature of water in K.

Heat transferred through the tube per second,

Q = Mass × Sp. Heat × Rise in temperature.

$$= \frac{1000 \times 4.2(T_{w2} - 303)}{60}$$

$$= 70(T_{w2} - 303) \text{kJ/s} \text{ ------- (i)}$$

We also know that

$$Q = \frac{2\pi \ell k(T_1 - T_2)}{ln\left(\frac{r_2}{r_1}\right)}$$

$$= \frac{2\pi \times 2 \times 67.2\left[873 - \frac{(303 + T_{w2})}{2}\right]}{ln\left(\frac{0.03}{0.025}\right)}$$

$$= 4632\left[873 - \frac{(303 + T_{w2})}{2}\right]$$

$$= 4632\left(721.5 - \frac{T_{w2}}{2}\right)$$

$$= 4.632\left(721.5 - \frac{T_{w2}}{2}\right) kJ/s \text{ ------ (ii)}$$

Equating equations (i) and (ii)

$$70(T_{w2} - 303) = 4.632\left(721.5 - \frac{T_{w2}}{2}\right) kJ/s$$

$$T_{w2} = 339.5 K = 66.5^0 C$$

∴ Rise in temperature

$$= T_{w2} - T_{w1} = 66.5 - 30 = 36.5^0 C$$

A spherical shaped vessel of 1.2 m diameter is 100 mm thick. Find the rate of heat leakage, if the temperature difference between the inner and outer surface is 200^oC. Thermal conductivity of the material of sphere is 0.3 kJ/m-h^oC.

Sol: Given,

Outside diameter of sphere $d_2 = 1.2$ m

$r_2 = 0.6$ m

Inside diameter of sphere

$= 1.2 - 2 \times 0.100 = 1$m

$r_1 = 0.5$ m

$t_1 - t_2 = 200^0C, \quad k = 0.3$ kJ/m^{0}C

Heat transfer in hollow sphere is given by,

$$Q = \frac{4\pi k r_1 r_2 (t_1 - t_2)}{(r_2 - r_1)}$$

$$= \frac{4\pi \times 0.3 \times 0.5 \times 0.6 \times 200}{0.100} = 2262 \text{kJ/h}$$

A cylinder having its diameter 30 cm and length 60 cm, has hemispherical ends, giving an overall length of 90 cm. The cylinder which is maintained at a steady temperature of 60^{o}C is covered to a depth of 5 cm with lagging which has a coefficient of conductivity of 0.14 W/mK. Calculate the rate of heat loss if the outer surface of lagging is at 30^{o}C.

Sol: Rate of heat loss from cylindrical portion

$$Q = \frac{2\pi \ell k(t_1 - t_2)}{\ln(r_2 / r_1)}$$

$$= \frac{2 \times \pi \times 0.6 \times 0.14(60 - 30)}{\ln(20/15)} = 55\ \text{W}$$

Rate of heat loss from spherical portion

$$Q = \frac{4\pi k r_1 r_2 (t_1 - t_2)}{(r_2 - r_1)}$$

$$= \frac{4\pi \times 0.14 \times 0.15 \times 0.20(60 - 30)}{0.20 - 0.15}$$

$$= 31.667\ \text{W}$$

Hence total heat loss

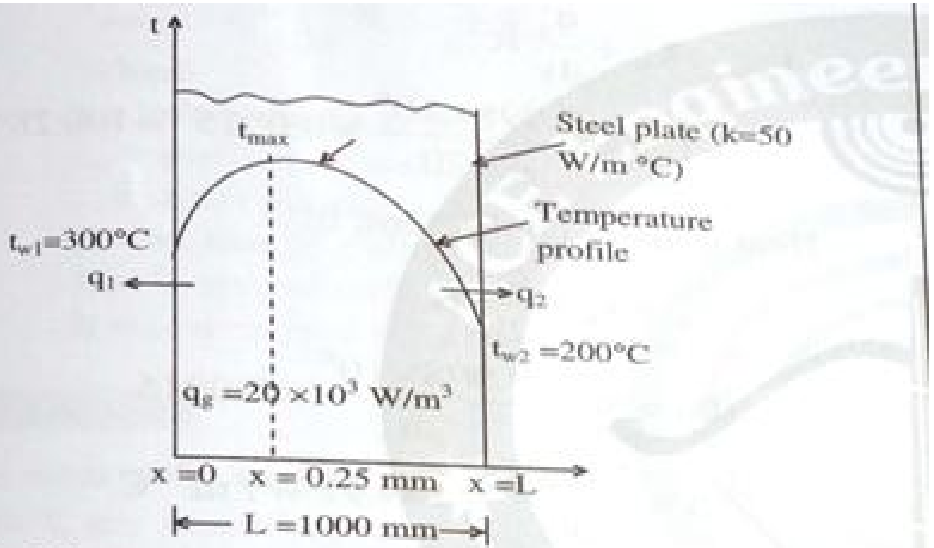
$$= 55 + 31.66 = 86.667\ \text{W}$$

The temperature on the two surfaces of a 1000 mm thick steel plate, (k=50 W/m^{o}C) having a uniform volumetric heat generation of 20x 10^3 W/m^3, are 300^{o}C and 200^{o}C. Determine the following:

i) the temperature distribution across the plate

ii) the value and position of the maximum temperature, and

iii) the flow of heat from each surface of the plate



Sol: Given,

$$L = 1000\text{mm} = 1 \text{ m}$$
$$t_{w1} = 300°C,\ t_{w2} = 200°C$$
$$q_g = 20 \times 10^6 \text{ W/m}^3;$$
$$k = 50 \text{ W/m °C}$$

(i) The temperature distribution, when both the surfaces of the wall have different temperatures, is given by

$$t = t_{w1} + \left(\frac{q_g}{2k}(L - x) + \frac{(t_{w2} - t_{w1})}{L}\right)x$$

Substituting the given values

$$t = 300 + \left(\frac{20000}{2 \times 50}(1 - x) + \frac{(200 - 300)}{1}\right)x$$
$$= 300 + (200(1 - x) - 100)\ x$$
$$t = 300 + 100x - 200\ x^2$$

(ii) In order to determine the position of maximum temperature, differentiating the above expression and equating it to zero, we obtain

$$\frac{dt}{dx} = 100 - 400\,x = 0$$

$$\therefore\ x = \frac{100}{400} = 0.25 \text{ m or } 25 \text{ mm}$$

The maximum temperature is

$$t_{max} = 300 + 100 \times 0.25 - 200 \times 0.25^2$$
$$= 312.5°C$$

(iii) The flow of heat from each surface of the plate, q_1, q_2:

The heat flow at the left face $(x = 0)$

$$q_1 = -k\,A\left(\frac{dt}{dx}\right)_{x=0}$$
$$= -50 \times 1 \times (100 - 400\,x)_{x=0}$$
$$= -5000 \text{ W/m}^2$$

The negative sign signifies that the heat flo at the left face is in a direction opposite to th of measurement of the distance,

$$q_2 = 50 \times 1 \times (100 - 400\,x)_{x=1}$$
$$= 50 \times 1 \times (100 - 400 \times 1)$$
$$= -15000 \text{ W/m}^2$$

A 3 mm diameter wire (k=20 W/m°C, resistivity, ρ= 10×10^{-8} Ω m) 100 m long has a voltage of 100 V impressed on it. The outer surface of the wire is maintained at 100°C. Calculate the centre temperature of the wire. If the heated wire is submerged in a fluid maintained at 50°C, find the heat transfer coefficient on the surface of the wire.

Sol: Given, Radius of wire,

$$R = 1.5\,\text{mm} = 0.0015\,\text{m}$$

Length of the wire, $L = 100$ m

Voltage impressed $= 100$ V

Thermal conductivity, $k = 20$ W/m °C

Resistivity, $\rho = 10 \times 10^{-8}\ \Omega\text{m}$

The temperature of the wire, $t_w = 100$ °C

Fluid temperature, $t_a = 50$ °C

Resistance of the wire, $R = \dfrac{\rho L}{A}$

$$= \frac{10 \times 10^{-8} \times 100}{\pi \times 0.0015^2} = 1.415\,\Omega$$

Rate of heat generation,

$$Q_g = VI = \frac{V^2}{R} = \frac{100^2}{1.415} = 7067\,W$$

$\therefore$ Rate of heat generation per unit volume

$$q_g = \frac{Q_g}{AL}$$

$$= \frac{7067}{\pi \times 0.0015^2 \times 100} = 9.998 \times 10^6\ W/m^3$$

Centre temperature is given by

$$t_{max} = t_w + \frac{q_g}{4k} R^2$$

$$= 100 + \frac{9.998 \times 10^6}{4 \times 20} \times 0.0015^2 = 100.28°C$$

Heat transfer coefficient, h:

$$t_w = t_a + \frac{q_g}{2h}.R$$

$$100 = 50 + \frac{9.998 \times 10^6}{2h} \times 0.0015$$

$$h = \frac{7498.5}{50} = 149.97\ W/m^2\,°C$$

The meat rolls of 25 mm diameter having k=1 W/m^{o}C are heated up with the help of microwave heating for roasting. The centre temperature of the rolls in maintained at 100^{o}C when the surrounding temperature is 30^{o}C. The heat transfer coefficient on the surface of the meat roll is 20 W/m^{o}C. Find the microwave heating capacity required in W/m^3.

ol: Given, $R = 0.0125\,m$, $k = 1 W/m°C$;

$t_{max} = 100°C$, $t_a = 30°C$, $h = 20\ W/m^2°C$.

Microwave heating capacity, q_g:

The maximum temperature occurs at the centre and is given by

$$t_{max} = t_a + \frac{q_g}{2h} \times R + \frac{q_g}{4k} \times R^2$$

$$100 = 30 + \frac{q_g \times 0.0125}{2 \times 20} + \frac{q_g \times 0.0125^2}{4 \times 1}$$

$$q_g = \frac{(100 - 30)}{(0.0003125 + 0.00003906)}$$

$$= 1.991 \times 10^5\ W/m^3$$