The global coordinates of the four corners of a linear quadratic element are given by:

$(x_1, y_1) = (6, 9)$ mm, $(x_2, y_2) = (2, 7)$ mm $(x_3, y_3) = (3, 10)$ mm in and $(x_4, y_4) = (10, 6)$ mm Find the global coordinates corresponding to the natural coordinates $\xi = -0.75$ and $\eta = 0.5$

Solution :-

$$N_1 = \frac{(1-\underbrace{\xi}_{(1.75)})(1-\underbrace{\eta}_{(0.5)})}{4} \qquad N_2 = \frac{(1+\underbrace{\xi}_{(0.25)})(1-\underbrace{\eta}_{(0.5)})}{4}$$

$$N_3 = \frac{(1+\underbrace{\xi}_{(0.25)})(1+\underbrace{\eta}_{(1.5)})}{4} \qquad N_4 = \frac{(1-\underbrace{\xi}_{(1.75)})(1+\underbrace{\eta}_{(1.5)})}{4}$$

$N_1 = 0.218 \qquad N_2 = 0.031$

$N_3 = 0.093 \qquad N_4 = 0.656$

$$x = N_1 x_1 + N_2 x_2 + N_3 x_3 + N_4 x_4$$

$$y = N_1 y_1 + N_2 y_2 + N_3 y_3 + N_4 y_4$$

$$x = (0.218)(6) + (0.031 \times 2) + (0.093 \times 3) + (0.656 \times 10)$$

$$= 1.308 + 0.062 + 0.279 + 6.56$$

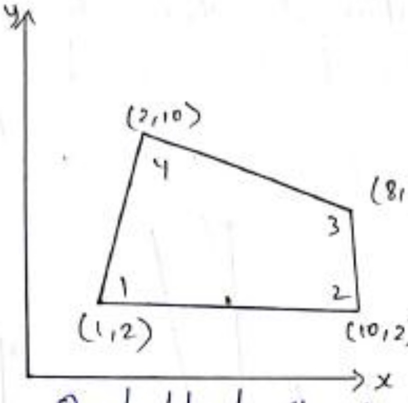
$$x = 8.209$$

$$y = (0.218 \times 9) + (0.031 \times 7) + (0.093 \times 10) + (0.656 \times 6)$$

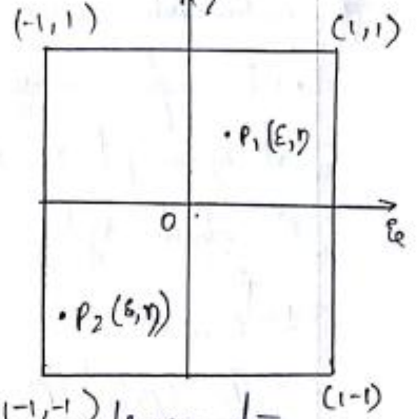
$$= 1.917 + 0.217 + 0.93 + 3.936$$

$$y = 7$$

1/1



Quadrilateral Element



Isoparametric Rectangular Element (or) Master Element

Substitute η, ε, $x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4$ values

$$[J] = \frac{1}{4}\begin{bmatrix} -(1+0.57735) & (1+0.57735) & (1-0.57735) & -(1-0.57735) \\ -(1-0.57735) & -(1-0.57735 & (1-0.57735) & (1-0.57735) \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 10 & 2 \\ 8 & 6 \\ 2 & 10 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} -1.57735 & 1.57735 & 0.4425 & -0.4225 \\ -0.4225 & -0.4225 & 0.4425 & 0.4225 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 10 & 2 \\ 8 & 6 \\ 2 & 10 \end{bmatrix}$$

Now

$$[J] = \begin{bmatrix} 4.19 & -0.42 \\ -0.69 & 2.42 \end{bmatrix} = 9.83$$

We know that strain - displacement matrix for quadrilateral element is given by

$$[B] = \frac{1}{|J|}\begin{bmatrix} J_{22} & J_{12} & 0 & 0 \\ 0 & 0 & -J_{21} & J_{11} \\ -J_{21} & J_{11} & J_{22} & -J_{12} \end{bmatrix} \times \frac{1}{4}\begin{bmatrix} -(1-\eta) & 0 & (1-\eta) & 0 & (1+\eta) & 0 & -1+\eta & 0 \\ -(1-\varepsilon) & 0 & -(1+\varepsilon) & 0 & (1+\varepsilon) & 0 & 1-\varepsilon & 0 \\ 0 & -(1-\eta) & 0 & (1+\eta) & 0 & (1+\eta) & 0 & -(1+\eta) \\ 0 & -(1-\varepsilon) & 0 & (1+\varepsilon) & 0 & (1+\varepsilon) & 0 & (1-\varepsilon) \end{bmatrix} \rightarrow (2)$$

Substitute J_{11}, J_{12}, J_{21}, J_{22}, $|J|$, η & ε value in eqn (2)

$$[B] = \frac{1}{9.83}\begin{bmatrix} 2.42 & 0.42 & 0 & 0 \\ 0 & 0 & 0.683 & 4.19 \\ 0.69 & 4.19 & 2.42 & 0.42 \end{bmatrix} \times \frac{1}{4}\begin{bmatrix} -1.58 & 0 & 1.58 & 0 & 0.42 & 0 & -1.58 & 0 \\ -0.42 & 0 & -0.42 & 0 & 1.58 & 0 & 0.42 & 0 \\ 0 & -1.58 & 0 & 1.58 & 0 & 0.42 & 0 & -0.42 \\ 0 & -0.42 & 0 & -0.42 & 0 & 1.58 & 0 & 0.42 \end{bmatrix}$$

$$= \frac{1}{4 \times 9.83}\begin{bmatrix} -4 & 0 & 3.15 & 0 & 1.7 & 0 & -0.84 & 0 \\ 0 & -2.84 & 0 & -5.52 & 0 & 6.89 & 0 & 1.47 \\ -2.84 & -4 & -5.52 & 3.15 & 6.89 & 1.7 & 1.471 & -0.84 \end{bmatrix}$$

Result

1. Jacobian matrix $[J] = \begin{bmatrix} 4.19 & -0.42 \\ -0.69 & 2.42 \end{bmatrix}$

2. Strain - displacement matrix,

$$[B] = \frac{1}{4 \times 9.83}\begin{bmatrix} -4 & 0 & 3.15 & 0 & 1.7 & 0 & -0.84 & 0 \\ 0 & -2.84 & 0 & -5.52 & 0 & 6.89 & 0 & 1.47 \\ -2.84 & -4 & -5.52 & 3.15 & 6.89 & 1.7 & 1.471 & -0.84 \end{bmatrix}$$

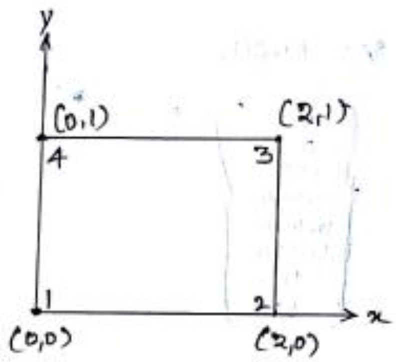
A four rectangular element is shown in fig.

(i) Determine the following :

1.) Jacobian matrix

2.) Strain - Displacement matrix

3.) Element stresses



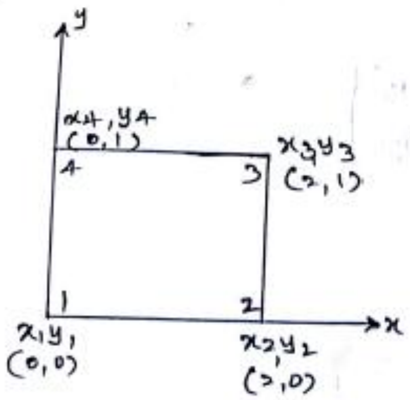
Take $E = 2 \times 10^5$ N/mm² $\nu = 0.25$

$u = [0, 0, 0.003, 0.004, 0.006, 0.004, 0, 0]^T$

$\xi = 0$; $\eta = 0$

Assume plane stress condition

Given :



1/7

Cartesian co-ordinates of point 1, 2, 3 and 4

$x_1 = 0; \quad y_1 = 0$

$x_2 = 2; \quad y_2 = 0$

$x_3 = 2; \quad y_3 = 1$

$x_4 = 0; \quad y_4 = 1$

Young's modulus, $E = 2 \times 10^5 \ N/m^2$

Poisson's ratio, $\nu = 0.25$

Displacements, $u = \begin{Bmatrix} 0 \\ 0 \\ 0.003 \\ 0.004 \\ 0.006 \\ 0.004 \\ 0 \\ 0 \end{Bmatrix}$

Natural co-ordinates, $\xi = 0, \eta = 0$

To find : 1.) Jacobian matrix, J

2.) strain – Displacement matrix [B],

3.) Element stress, σ

Sol:

$$[J] = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

where ,

2/7

$$J_{11} = \frac{1}{4}\left[-(1-\eta)x_1 + (1-\eta)x_2 + (1+\eta)x_3 - (1+\eta)x_4\right] \rightarrow (1)$$

$$J_{12} = \frac{1}{4}\left[-(1-\eta)y_1 + (1-\eta)y_2 + (1+\eta)y_3 - (1+\eta)y_4\right] \rightarrow (2)$$

$$J_{21} = \frac{1}{4}\left[-(1-\xi)x_1 - (1+\xi)x_2 + (1+\xi)x_3 + (1-\xi)x_4\right] \rightarrow (3)$$

$$J_{22} = \frac{1}{4}\left[-(1-\xi)y_1 - (1+\xi)y_2 + (1+\xi)y_3 + (1-\xi)y_4\right] \rightarrow (4)$$

Substitude $x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4$

$$(1) \Rightarrow J_{11} = \frac{1}{4}\left[0+2+2-0\right] \Rightarrow \boxed{J_{11} = 1}$$

$$(2) \Rightarrow J_{12} = \frac{1}{4}\left[0+0+1-1\right] \Rightarrow \boxed{J_{12} = 0}$$

$$(3) \Rightarrow J_{21} = \frac{1}{4}\left[0-2+2+0\right] \Rightarrow \boxed{J_{21} = 0}$$

$$(4) \Rightarrow J_{22} = \frac{1}{4}\left[-0-0+1+1\right] \Rightarrow \boxed{J_{22} = 0.5}$$

Jacobian matrix:

$$[J] = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

$$[J] = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix} \rightarrow (5)$$

$$|J| = 1 \times 0.5 - 0$$

$$\boxed{|J| = 0.5} \rightarrow (6)$$

3/~~7~~

We know that, strain-Displacement matrix for quadrilateral element is

$$\Rightarrow [B] = \frac{1}{|J|}\begin{bmatrix} J_{22} & -J_{12} & 0 & 0 \\ 0 & 0 & -J_{21} & J_{11} \\ -J_{21} & J_{11} & J_{22} & -J_{12} \end{bmatrix} \times \frac{1}{4}\begin{bmatrix} -(1-\eta) & 0 & (1+\eta) & 0 & (1+\eta) & 0 & -(1+\eta) & 0 \\ -(1-\xi) & 0 & -(1+\xi) & 0 & (1+\xi) & 0 & (1-\xi) & 0 \\ 0 & -(1-\eta) & 0 & (1-\eta) & 0 & (1+\eta) & 0 & -(1+\eta) \\ 0 & -(1-\xi) & 0 & -(1+\xi) & 0 & (1+\xi) & 0 & (1-\xi) \end{bmatrix}$$

$$= \frac{1}{0.5 \times 4}\begin{bmatrix} -0.5 & 0 & 0.5 & 0 & 0.5 & 0 & -0.5 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 & 0 & 1 \\ -1 & -0.5 & -1 & 0.5 & 1 & 0.5 & 1 & -0.5 \end{bmatrix}$$

$$= \frac{0.5}{0.5 \times 4}\begin{bmatrix} -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -2 & 0 & -2 & 0 & 2 & 0 & 2 \\ -2 & -1 & -2 & 1 & 2 & 1 & 2 & -1 \end{bmatrix} \rightarrow (7)$$

4/4

We know that,

Element Stress, $\sigma = [D][B]\{u\} \rightarrow (8)$

For plane stress Condition,

$$[D] = \frac{E}{1-v^2}\begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1-v}{2} \end{bmatrix}$$

$$= \frac{2\times10^5}{1-(0.25)^2}\begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & \frac{1-0.25}{2} \end{bmatrix}$$

$$= 213.33\times10^3\begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & 0.375 \end{bmatrix}$$

$$= 213.33\times10^3\times0.25\begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

$$[D] = 53.333\times10^3\begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix} \rightarrow (9)$$

$$\Rightarrow \{\sigma\} = 53.333\times10^3\begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix} \times$$

$$0.25\begin{bmatrix} -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -2 & 0 & -2 & 0 & 2 & 0 & 2 \\ -2 & -1 & -2 & 1 & 2 & 1 & 2 & -1 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ 0.003 \\ 0.004 \\ 0.006 \\ 0.004 \\ 0 \\ 0 \end{Bmatrix}$$

5/7

$$= 53.333\times10^3\times0.25\begin{bmatrix}-4 & -2 & 4 & -2 & -4 & 2\\ -1 & -8 & 1 & -8 & -1 & 8\\ -3 & -1.5 & -3 & 1.5 & 3 & -1.5\end{bmatrix}\begin{Bmatrix}0\\0\\0.003\\0.004\\0.006\\0.004\\0\\0\end{Bmatrix}$$

$$= 13.333\times10^3\begin{Bmatrix}0+0+(4\times0.003)+(-2\times0.004)+(4\times0.006)+(2\times0.004)+0+0\\ 0+0+(1\times0.003)+(-8\times0.004)+(1\times0.006)+(8\times0.004)+0+0\\ 0+0+(-3\times0.003)+(1.5\times0.004)+(3\times0.006)+(1.5\times0.004)+0+0\end{Bmatrix}$$

$$\{\sigma\} = 13.333\times10^3\begin{Bmatrix}0.036\\0.009\\0.021\end{Bmatrix}$$

$$\{\sigma\} = \begin{Bmatrix}480\\120\\280\end{Bmatrix} N/m^2$$

Result:

1.) Jacobian matrix, $[J] = \begin{bmatrix}1 & 0\\ 0 & 0.5\end{bmatrix}$

2.) Strain–Displacement matrix, $[B]$ $= 0.25\begin{bmatrix}-1 & 0 & 1 & 0 & 1 & 0 & -1 & 0\\ 0 & -2 & 0 & -2 & 0 & 2 & 0 & 2\\ 2 & -1 & -2 & 1 & 2 & 1 & 2 & -1\end{bmatrix}$

3.) Stress $\{\sigma\} = \begin{Bmatrix}480\\120\\280\end{Bmatrix} N/m^2$

6/7

Evaluate the integral, $I = \int_{-1}^{1} [x^2 + \cos(x/2)]\,dx$ using three point Gaussian quadrature and compare with exact solution.

Given :

Integral $I = \int_{-1}^{1} x^2 + \cos\left(\frac{x}{2}\right) dx$

$f(x) = x^2 + \cos\left(\frac{x}{2}\right)$

To find : Evaluate the integral by using 3 point Gaussian quadrature and compare with exact solution.

Soln : For three point Gaussian quadrature,

$x_1 = \sqrt{\frac{3}{5}} \Rightarrow 0.774596669$

$x_2 = 0$

$x_3 = -\sqrt{\frac{3}{5}} \Rightarrow -0.77459669$

$w_1 = 5/9 = 0.555555$

$w_2 = 8/9 = 0.888888$

$w_3 = 5/9 = 0.555555$

We know that

$f(x) = x^2 + \cos\left(\frac{x}{2}\right)$

$\Rightarrow f(x_1) = x_1^2 + \cos\left(\frac{x_1}{2}\right)$

$= (0.77459669)^2 + \cos\left(\frac{0.77459669}{2}\right)$ rad

$f(x) = 1.5259328$

$w_1 f(x_1) = 0.555555 \times 1.5259328$

$w_1 f(x_1) = 0.8477396$

1/2

$f(x_2) = x_2^2 + \cos\left(\frac{x_2}{2}\right)$

$= (0)^2 + \cos\left(\frac{0}{2}\right)$ rad

$f(x_2) = 1$

$w_2 f(x_2) = 0.888888 \times 1$

$w_2 f(x_2) = 0.888888$

$f(x_3) = x_3^2 + \cos\left(\frac{x_3}{2}\right)$

$= (-0.774596669)^2 + \cos\left(\frac{-0.774596669}{2}\right)$ rad

$f(x_3) = 1.5259328$

$w_3 f(x_3) = 0.555555 \times 1.5259328$

$w_3 f(x_3) = 0.8477396$

Add equ ①, ② & ③

$w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3) = 0.8477396 + 0.888889 + 0.8477396$

$$\int_{-1}^{1} \left[x^2 + \cos\left(\frac{x}{2}\right)\right] dx = 2.58436$$

Exact solution:

$$\int_{-1}^{1} \left[x^2 + \cos\left(\frac{x}{2}\right)\right] dx = \left[\frac{x^3}{3}\right]_{-1}^{+1} + \left[\frac{\sin\left(\frac{x}{2}\right)}{\left(\frac{1}{2}\right)}\right]_{-1}^{1}$$

$$= \frac{1}{3}\left[1^3 - (-1)^3\right] + 2\left[\sin\left(\frac{1}{2}\right) - \sin\left(\frac{-1}{2}\right)\right]_{rad}$$

$$= 2.58436$$

Result:

1. $\int_{-1}^{1} \left[x^2 + \cos\left(\frac{x}{2}\right)\right] dx = 2.58436$

[By three point Gauss quadrature]

2. $\int_{-1}^{1} \left[x^2 + \cos\left(\frac{x}{2}\right)\right] dx = 2.58436$

[By exact solution]

2/2

Integrate the function $f(r) = 1 + r + r^2 + r^3$ between the limits -1 and +1 using

i) exact method

ii) Gauss integration method and compare the two results

Given:

function $f(r) = 1 + r + r^2 + r^3$

To find:

Evaluate the integral by using Gauss integration method and compare with exact method.

Solution:

We know that the given integrand is a polynomial of order 3

so, $2n - 1 = 3$

$2n = 4$

$n = 2$

We should use two sampling points for two point Gaussian quadrature.

$r_1 = +\sqrt{\frac{1}{3}} = 0.577350269$ | $w_1 = 1$

$r_2 = -\sqrt{\frac{1}{3}} = -0.577350269$ | $w_2 = 1$

$f(r) = 1 + r + r^2 + r^3$

$f(r_1) = 1 + r_1 + r_1^2 + r_1^3$

$= 1 + 0.577350269 + (-0.577350269)^2 + (0.577350269)^3$

$f(r_1) = 2.1031336$

$w_1 f(r_1) = 1 \times 2.1031336$

$w_1 f(r_1) = 2.1031336 \quad — (1)$

$f(r_2) = 1 + r_2 + r_2^2 + r_2^3$

$= 1 + (-0.577350269) + (-0.577350269)^2 + (-0.577350269)^3$

$f(r_2) = 0.5635329$

$w_2 f(r_2) = 1 \times 0.5635329$

$w_2 f(r_2) = 0.563529. \quad — (2)$

Adding (1) and (2)

$w_1 f(r_1) + w_2 f(r_2) = 2.1031336 + 0.5635326$

$= 2.666666$

$= \int_{-1}^{1} (1 + r + r^2 + r^3) = 2.666666$

Exact method:

$\int_{-1}^{1} (1 + r + r^2 + r^3) dr = \left(r + \frac{r^2}{2} + \frac{r^3}{3} + \frac{r^4}{4} \right)_{-1}^{1}$

$= (r)_{-1}^{1} + \frac{1}{2} (r^2)_{-1}^{1} + \frac{1}{3} (r^3)_{-1}^{1} + \frac{1}{4} (r^4)_{-1}^{1}$

$= (1 - (-1)) + \frac{1}{2} ((1)^2 - (-1)^2) + \frac{1}{3} (1^3 - (-1)^3) + \frac{1}{4} (1^4 - (-1)^4)$

$= 2 + \frac{1}{2}(0) + \frac{1}{3}(1+1) + \frac{1}{4}(0)$

$\int_{-1}^{1} (1 + r + r^2 + r^3) dr = 2.666666$

Result:

1. $\int_{-1}^{1} (1 + r + r^2 + r^3) dr = 2.666666$ (By Gauss integration)
2. $\int_{-1}^{1} (1 + r + r^2 + r^3) dr = 2.666666$ (By exact method)

Evaluate $I = \int_{-1}^{1}\left[3e^x + x^2 + \frac{1}{x+2}\right]dx$ using one point and two point Gauss quadrature. Compare with exact solution

Given:

Integral $I = \int_{-1}^{1}\left[3e^x + x^2 + \frac{1}{x+2}\right]dx$

$f(x) = 3e^x + x^2 + \frac{1}{x+2}$

To find: 1. Evaluate the integral by using one point and two point

2. Compare with exact solution

Soln:

$x_1 = 0$; $w_1 = 2$

$f(x) = 3e^x + x^2 + \frac{1}{x+2}$

$f(x_1) = 3e^{x_1} + x_1^2 + \frac{1}{x_1+2}$

$= 3e^0 + 0 + \frac{1}{0+2}$

$f(x_1) = 3.5$

$w_1 f(x_1) = 2 \times 3.5$

$w_1 f(x_1) = 7$

$\int_{-1}^{1}\left[3e^x + x^2 + \frac{1}{(x+2)}\right]dx = 7$ for one point Gauss quadrature

For two point Gauss quadrature

$x_1 = +\sqrt{\frac{1}{3}} = 0.577350269$

$x_2 = -\sqrt{\frac{1}{3}} = 0.577350269$

$w_1 = 1$

$w_2 = 1$

$f(x) = 3e^x + x^2 + \frac{1}{(x+2)}$

$f(x_1) = 3e^{x_1} + x_1^2 + \frac{1}{(x_1+2)}$

$= 3e^{(0.577350269)} + (0.577350269)^2 + \frac{1}{0.577350269+2}$

$f(x_1) = 6.065265$

$w_1 f(x_1) = 1 \times (6.065265)$

1/2

$w_1 f(x_1) = 6.065265$

$f(x_2) = 3e^{x_2} + x_2^2 + \frac{1}{x_2+2}$

$= 3e^{-0.577350269} + (-0.577350269)^2 + \frac{1}{-0.577350269+2}$

$f(x_2) = 2.7203987$

$w_2 f(x_2) = 1 \times 2.7203987$

$w_2 f(x_2) = 2.7203987$

Adding ① & ②

$\Rightarrow w_1 f(x_1) + w_2 f(x_2) = 6.065265 + 2.7203987 = 8.7856$

$\int_{-1}^{1} \left(3e^x + x^2 + \frac{1}{x+2}\right) dx = 8.7856$ for two point Gauss quadrature

Exact solution:

$I = \int_{-1}^{1} \left(3e^x + x^2 + \frac{1}{x+2}\right) dx$

$= 3\left[e^x\right]_{-1}^{1} + \left[\frac{x^3}{3}\right]_{-1}^{1} + \left[\ln(x+2)\right]_{-1}^{1}$

$= 3\left[e^1 - (e^{-1})\right] + \frac{1}{3}\left[1^3 - (-1)^3\right] + \left[\ln(1+2) - \ln(-1+2)\right]$

$= 3\left[2.718 - 0.36773\right] + \frac{1}{3}\left[1+1\right] + \ln(3) - \ln(1)$

$\int_{-1}^{1} \left(3e^x + x^2 + \frac{1}{x+2}\right) dx = 8.8158$

Result:

1. One point Gauss quadrature

$\int_{-1}^{1} \left(3e^x + x^2 + \frac{1}{x+2}\right) dx = 7$

2. Two point Gauss quadrature

$\int_{-1}^{1} \left(3e^x + x^2 + \frac{1}{x+2}\right) dx = 8.7856$

3. Exact solution

$\int_{-1}^{1} \left(3e^x + x^2 + \frac{1}{x+2}\right) dx = 8.8158$

2/1

1. Evaluate the integral $I = \int_{-1}^{1}\int_{-1}^{1} (2x^2+3xy+4y^2)\,dx\,dy$. using Gauss integration.

Given:

integral $I = \int_{-1}^{1}\int_{-1}^{1} (2x^2+3xy+4y^2)\,dx\,dy$

$f(x,y) = (2x^2+3xy+4y^2)$

To find:

evaluate the integral by using Gauss integration

Solution: we know that the given integral is a polynomial of order 2. so, for exact integration

$2n-1 = 2$

$n = 1.5 \simeq 2$

$n = 2.$

we should use two sampling points for two point Gaussian quadrature.

$x_1 = 0.57735 \quad y_1 = 0.57735$

$x_2 = 0.57735 \quad y_2 = -0.57735$

$w_1 = 1$

$w_2 = 1$

for two point scheme, the above equation can be written as.

$$\int_{-1}^{1}\int_{-1}^{1} f(x,y)\,dx\,dy = w_1^2 f(x_1,y_1) + w_1 w_2 (x_1,y_2) + w_2 w_1 f(x_2,y_1) + w_2^2 f(x_2 y_2) \quad -(1)$$

we know that:

$f(x,y) = (2x^2+3xy+4y^2)$

$w_1^2 f(x_1,y_1) = w_1^2(2x_1^2+3x_1y_1+4y_1^2)$

$\frac{1}{3}$

$= 1^2 \left(2(0.57735)^2 + 3(0.57735)(0.57735) + 4(0.57735)^2\right)$

$w_1^2 f(x_1 y_1) = 3 \quad -②$

$w_1 w_2 f(x_1, y_2) = w_1 w_2 \left(2x_1^2 + 3x_1 y_2 + 4y_2^2\right)$

$= 1 \times 1 \left(2(0.57735)^2 + 3(0.57735)(-0.57735) + 4(-0.57735)^2\right) \quad -③$

$w_1 w_2 f(x_1, y_2) = 1$

$w_2 w_1 f(x_2 y_1) = w_2 w_1 \left(2x_2^2 + 3x_2 y_1 + 4y_1^2\right)$

$= 1 \times 1 \left(2(-0.57735)^2 + 3(-0.57735)(0.57735) + 4(0.57735)^2\right) \quad -④$

$w_2 w_1 f(x_2 y_1) = 1$

$w_2^2 f(x_2 y_2) = w_2^2 \left(2x_2^2 + 3x_2 y_2 + 4y_2^2\right)$

$= 1^2 \left(2(-0.57735)^2 + 3(-0.57735)(-0.57735) + 4(-0.57735)^2\right) \quad -⑤$

$w_2^2 f(x_2, y_2) = 3$

Substitute the equation (2) (3) (4) and (5) in equation (1)

$$\int_{-1}^{1}\int_{-1}^{1} (2x^2 + 3xy + 4y^2)\,dx\,dy = 3 + 1 + 1 + 3 = 8$$

$$\boxed{\int_{-1}^{1}\int_{-1}^{1} (2x^2 + 3xy + 4y^2)\,dx\,dy = 8}$$

Verification: The exact solution of integral is

$$\int_{-1}^{1}\int_{-1}^{1} (2x^2 + 3xy + 4y^2)\,dx\,dy$$

$\frac{2}{3}$

$$= \int_{-1}^{1} \left\{ \left[\frac{2}{3} x^3 + \frac{3}{2} y x^2 + 4y^2 x \right]_{-1}^{1} \right\} dy$$

$$= \int_{-1}^{1} \left\{ \frac{2}{3}(1+1) + \frac{3}{2} y (1-1) + 4y^2 (1+1) \right\} dy$$

$$= \int_{-1}^{1} \left(\frac{4}{3} + 8y^2 \right) dy$$

$$= \left[\frac{4}{3} y + \frac{8}{3} y^3 \right]_{-1}^{1}$$

$$= \frac{4}{3}(1+1) + \frac{8}{3}(1+1) = \frac{8}{3} + \frac{16}{3} = \frac{24}{3} = 8$$

$$\boxed{\int_{-1}^{1} \int_{-1}^{1} (2x^2 + 3xy + 4y^2)\, dx\, dy = 8.}$$

Result:

The integral $\int_{-1}^{1} \int_{-1}^{1} (2x^2 + 3xy + 4y^2)\, dx\, dy = 8.$

$\frac{3}{3}$