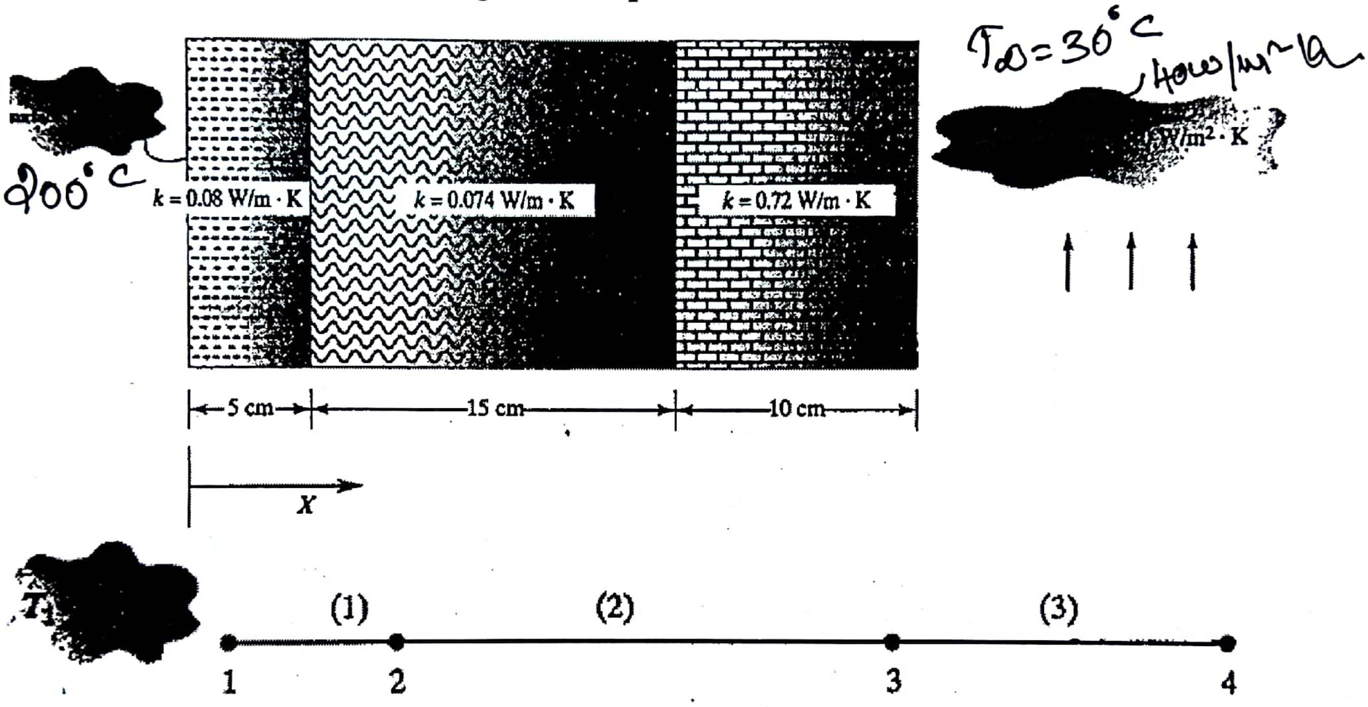
A wall of an industrial oven consists of three different materials, as depicted in figure 1.0. The first layer is composed of 5cm of insulating cement with a clay binder that has a thermal conductivity of 0.08W/m.K. The second layer is made from 15cm of 6-ply asbestos board with a thermal conductivity of 0.074W/m.K. The exterior consists of 10cm common brick with a thermal conductivity of 0.72W/m².K. The inside wall temperature of oven is 200 °C, and the outside air is 30°C with a convection coefficient of $40W/m^2.K$. Determine the temperature distribution along the composite wall.



A composite wall of an industrial oven.

Stiffness matrix for Element ①

$$K^{[1]} = \frac{K A}{L_1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{0.08 \times 1}{0.05}\begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix} = \begin{array}{cc} 1 & 2 \\ \begin{bmatrix} 1.6 & -1.6 \\ -1.6 & 1.6 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \end{array} \end{array} W/°C$$

Stiffness matrix for Element ②

$$K^{(2)} = \frac{K_2 A}{L_2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{0.074 \times 1}{0.15}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{array}{cc} 2 & 3 \\ \begin{bmatrix} 0.493 & -0.493 \\ -0.493 & 0.493 \end{bmatrix} & \begin{array}{c} 2 \\ 3 \end{array} \end{array} W/°C$$

For Element (3), including the boundary conditions.

$$[K]^{(3)} = \frac{k_3 A}{L_3}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & hA \end{bmatrix}$$

$$= \frac{0.72\times 1}{0.1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 40\times 1 \end{bmatrix}$$

$$= \begin{array}{cc} \begin{array}{cc} 3 & 4 \end{array} & \\ \begin{bmatrix} 7.2 & -7.2 \\ -7.2 & 47.2 \end{bmatrix} & \begin{array}{c} 3 \\ 4 \end{array} \end{array}$$

Thermal load/force vector.

$$\{F\}^{(1)} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} W \qquad F^{(2)} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} W$$

$$\{F\}^{(3)} = \begin{Bmatrix} 0 \\ hAT_f \end{Bmatrix} = \begin{Bmatrix} 0 \\ 40\times 1\times 30 \end{Bmatrix} = \begin{bmatrix} 0 \\ 1200 \end{bmatrix} W$$

Assembling elements, we obtain

$$[K]^{(G)} = \begin{bmatrix} 1.6 & -1.6 & 0 & 0 \\ -1.6 & 1.6+0.493 & -0.493 & 0 \\ 0 & -0.493 & 0.493+7.2 & -7.2 \\ 0 & 0 & -7.2 & 47.2 \end{bmatrix}$$

$$[F]^{(G)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1200 \end{bmatrix}$$

Apply the boundary condition at the inside furnace wall, we get:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1.6 & 2.093 & -0.493 & 0 \\ 0 & -0.493 & 7.693 & -7.2 \\ 0 & 0 & -7.2 & 47.2 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} = \begin{Bmatrix} 200 \\ 0 \\ 0 \\ 1200 \end{Bmatrix}$$

and solving the set of linear equations, we have the following results.

$$-1.6 \times T_1 + 2.093 T_2 - 0.493 T_3 = 0$$

$$2.093 T_2 - 0.493 T_3 = 320 \quad \text{--- (1)}$$

$$-0.493 T_2 + 7.693 T_3 - 7.2 T_4 = 0 \quad \text{--- (2)}$$

$$-7.2 T_3 + 47.2 T_4 = 1200 \quad \text{--- (3)}$$

$$\begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} = \begin{Bmatrix} 200 \\ 162.3 \\ 39.9 \\ 31.5 \end{Bmatrix} {}^\circ C$$