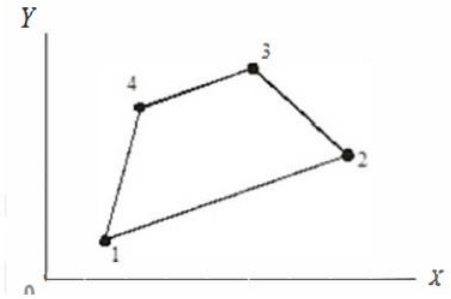
UNIT V- ISOPARAMETRIC FORMULATION

ISOPARAMETRIC ELEMENT: FOUR NODED **QUADRILATERAL**: SHAPE FUNCTION:



$$N_1 = \frac{1}{4}(1-\varepsilon)(1-\eta), \qquad N_2 = \frac{1}{4}(1+\varepsilon)(1-\eta)$$

$$N_3 = \frac{1}{4}(1+\varepsilon)(1+\eta), \qquad N_4 = \frac{1}{4}(1-\varepsilon)(1+\eta)$$

$$x = N_1x_1 + N_2x_2 + N_3x_3 + N_4x_4$$

$$y = N_1y_1 + N_2y_2 + N_3y_3 + N_4y_4$$

Jacobian Matrix

$$[J] = \begin{bmatrix} \frac{\partial x}{\partial \varepsilon} & \frac{\partial y}{\partial \varepsilon} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \qquad [J] = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

$$J_{11} = \frac{1}{4}\left[-(1-\eta)x_1 + (1-\eta)x_2 + (1+\eta)x_3 - (1+\eta)x_4\right]$$

$$J_{12} = \frac{1}{4}\left[-(1-\eta)y_1 + (1-\eta)y_2 + (1+\eta)y_3 - (1+\eta)y_4\right]$$

$$J_{21} = \frac{1}{4}\left[-(1-\varepsilon)x_1 - (1+\varepsilon)x_2 + (1+\varepsilon)x_3 + (1-\varepsilon)x_4\right]$$

$$J_{22} = \frac{1}{4}\left[-(1-\varepsilon)y_1 - (1+\varepsilon)y_2 + (1+\varepsilon)y_3 + (1-\varepsilon)y_4\right]$$

Strain displacement Matrix:

$$[B]=\frac{1}{|J|}\begin{bmatrix} J_{22} & -J_{12} & 0 & 0 \\ 0 & 0 & -J_{21} & J_{11} \\ -J_{21} & J_{11} & J_{22} & -J_{12} \end{bmatrix}\times\frac{1}{4}\begin{bmatrix} -(1-\eta) & 0 & (1-\eta) & 0 & (1+\eta) & 0 & -(1+\eta) & 0 \\ -(1-\varepsilon) & 0 & -(1+\varepsilon) & 0 & (1+\varepsilon) & 0 & (1-\varepsilon) & 0 \\ 0 & -(1-\eta) & 0 & (1-\eta) & 0 & (1+\eta) & 0 & -(1+\eta) \\ 0 & -(1-\varepsilon) & 0 & -(1+\varepsilon) & 0 & (1+\eta) & 0 & (1-\varepsilon) \end{bmatrix}$$

Stress Strain Relationship Matrix:
For Plane stress condition:

$$[D]=\frac{E}{1-\gamma^2}\begin{bmatrix} 1 & \gamma & 0 \\ \gamma & 1 & 0 \\ 0 & 0 & \frac{1-\gamma}{2} \end{bmatrix}$$

For Plane Strain condition:

$$[D]=\frac{E}{(1+\gamma)(1-2\gamma)}\begin{bmatrix} (1-\gamma) & \gamma & 0 \\ \gamma & (1-\gamma) & 0 \\ 0 & 0 & \frac{(1-2\gamma)}{2} \end{bmatrix}$$

E-Young's Modulus N/mm^2
γ - Poisson's ratio

Stiffness Matrix:
For Isoparametric Quadrilateral Element:

$$[K]=t\iint[B]^T[D][B]\partial x\partial y$$

For Natural Co-ordinates

$$\int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]\times|J|\times\partial\varepsilon\times\partial\eta$$

$[B]$ ⟶ Strain Displacement matrix
$[D]$ ⟶ Stress – Strain Relationship matrix
$|J|$ ⟶ Determinant of the Jacobian matrix
ε,η ⟶ Natural Co – Ordinates
t ⟶ Thickness of the element - mm
A ⟶ Area of the triangular element - mm^2