PERMUTATION AND COMBINATION

Permutation and combination are the ways to select certian objects from a group of objects to form subsets with or without replacement. It defines the various ways to arrange a certain group of data. When we select the data or objects from a certain group, it is said to be permutations, whereas the order in which they are represented is called combination.

What is Permutation?

In mathematics, **permutation relates to the act of arranging all the members of a set into some sequence or order.** In other words, if the set is already ordered, then the rearranging of its elements is called the process of permuting. Permutations occur, in more or less prominent ways, in almost every area of mathematics. They often arise when different orderings on certain finite sets are considered.

What is a Combination?

The **combination is a way of selecting items from a collection, such that (unlike permutations) the order of selection does not matter**. In smaller cases, it is possible to count the number of combinations. Combination refers to the combination of n things taken k at a time without repetition. To refer to combinations in which repetition is allowed, the terms k-selection or k-combination with repetition are often used.

Permutation and Combination Formulas

There are many formulas involved in permutation and combination concepts. The two key formulas are:

Permutation Formula

A permutation is the choice of r things from a set of n things without replacement and where the order matters.

$$^{n}P_{r} = (n!) / (n-r)!$$

Combination Formula

A combination is the choice of r things from a set of n things without replacement and where order does not matter.

$$_nC_r = \binom{n}{r} = \frac{_nP_r}{r!} = \frac{n!}{r!(n-r)!}$$

Difference Between Permutation and Combination

Permutation	Combination
Arranging people, digits, numbers, alphabets, letters, and colours	Selection of menu, food, clothes, subjects, team.
Picking a team captain, pitcher and shortstop from a group.	Picking three team members from a group.
Picking two favourite colours, in order, from a colour brochure.	Picking two colours from a colour brochure.
Picking first, second and third place winners.	Picking three winners.

Solved Examples of Permutation and Combinations

Example 1:

Find the number of permutations and combinations if n = 12 and r = 2.

Solution:

Given,n=12 r = 2

Using the formula given above:

Permutation:

${}^{n}P_{r} = (n!) / (n-r)! = (12!) / (12-2)! = 12! / 10! = (12 \times 11 \times 10!) / 10! = 132$

Combination:

$$ {}_{n}C_{r} = \frac{n!}{r!(n-r)!} $$
$$ \frac{12!}{2!(12-2)!} = \frac{12!}{2!(10)!} = \frac{12 \times 11 \times 10!}{2!(10)!} = 66 $$

Example 3:

In how many ways a committee consisting of 5 men and 3 women, can be chosen from 9 men and 12 women?

Solution:

Choose 5 men out of 9 men = 9C5 ways = 126 ways

Choose 3 women out of 12 women = 12C3 ways = 220 ways

Total number of ways = (126 x 220)= 27720 ways

The committee can be chosen in 27720 ways.

1.From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?
1.564   2.645 3.735 4.756 None of these

Answer: Option D

Explanation:

We may have (3 men and 2 women) or (4 men and 1 woman) or (5 men only).

$\therefore$ Required number of ways $= ({}^{7}C_{3} \times {}^{6}C_{2}) + ({}^{7}C_{4} \times {}^{6}C_{1}) + ({}^{7}C_{5})$

$$= \left(\frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times \frac{6 \times 5}{2 \times 1}\right) + ({}^{7}C_{3} \times {}^{6}C_{1}) + ({}^{7}C_{2})$$

$$= 525 + \left(\frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times 6\right) + \left(\frac{7 \times 6}{2 \times 1}\right)$$

$$= (525 + 210 + 21)$$

$$= 756.$$

2.In how many different ways can the letters of the word 'LEADING' be arranged in such a way that the vowels always come together?

- 360
- 480
- 720
- 5040
- None of these

Answer: Option ©

Explanation:

The word 'LEADING' has 7 different letters.

When the vowels EAI are always together, they can be supposed to form one letter.

Then, we have to arrange the letters LNDG (EAI).

Now, 5 (4 + 1 = 5) letters can be arranged in 5! = 120 ways.

The vowels (EAI) can be arranged among themselves in 3! = 6 ways.

∴ Required number of ways = (120 × 6) = 720.

Video Explanation: **https://youtu.be/WCEF3iW3H2c**

3.In how many different ways can the letters of the word 'CORPORATION' be arranged so that the vowels always come together?

- 810
- 1440
- 2880
- 50400
- 5760

Answer: Option Ⓓ

Explanation:

In the word 'CORPORATION', we treat the vowels OOAIO as one letter.

Thus, we have CRPRTN (OOAIO).

This has 7 (6 + 1) letters of which R occurs 2 times and the rest are different.

Number of ways arranging these letters = $\frac{7!}{2!}$ = 2520.

Now, 5 vowels in which O occurs 3 times and the rest are different, can be arranged

in $\frac{5!}{3!}$ = 20 ways.

∴ Required number of ways = (2520 × 20) = 50400.

Video Explanation: **https://youtu.be/o3fwMoB0duw**

4. Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

- 210
- 1050
- 25200
- 21400
- None of these

Answer: Option Ⓒ

Explanation:

Number of ways of selecting (3 consonants out of 7) and (2 vowels out of 4)

$= ({}^7C_3 \times {}^4C_2)$

$= \left(\frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times \frac{4 \times 3}{2 \times 1} \right)$

$= 210.$

Number of groups, each having 3 consonants and 2 vowels = 210.

Each group contains 5 letters.

Number of ways of arranging 5 letters among themselves $= 5!$

$= 5 \times 4 \times 3 \times 2 \times 1$

$= 120.$

$\therefore$ Required number of ways = (210 × 120) = 25200.

Video Explanation: **https://youtu.be/dm-8T8Si5lg**

5.

In how many ways can the letters of the word 'LEADER' be arranged?

72

144

360

720

None of these

Answer: Option Ⓒ

Explanation:

The word 'LEADER' contains 6 letters, namely 1L, 2E, 1A, 1D and 1R.

$\therefore$ Required number of ways $= \frac{6!}{(1!)(2!)(1!)(1!)(1!)} = 360.$

Video Explanation: **https://youtu.be/2_2QukHfkYA**