SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

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Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF AUTOMOBILE ENGINEERING

19AUB303 – Finite Element Methods and Analysis

III YEAR / VI SEM

UNIT – 3 FORMULATION OF ELEMENT CHARACTERISTIC MATRICES

AND VECTORS FOR THERMAL PROBLEMS

Topic - 6 - Problems on 2D Elements (Axisymmetric Elements)

2D Element - Problem

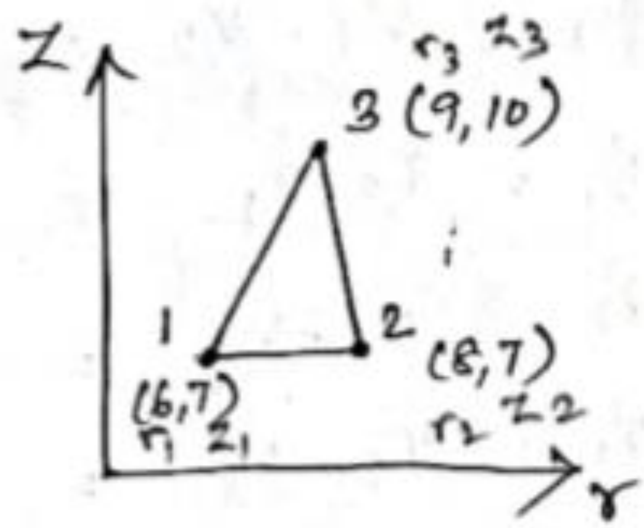
Calculate the element stiffness matrix and the thermal force vector for the axisymmetric element shown below. The Element experiences a 150 C increase in temperature.

Take,

$\alpha = 10 \times 10^{-6} / {}^{0}C$

$E = 2 \times 10^{5}$ N/mm^{2}

$\mu = 0.25$



To Find:

(i) Element stiffness matrix [K]

(ii) Thermal force vector $\{F\}_t$

Solution -

$$[K] = 2\pi A r \, [B]^T [D] [B]$$

$$[B] = \frac{1}{2A}\begin{bmatrix} \beta_1 & 0 & \beta_2 & 0 & \beta_3 & 0 \\ \frac{\alpha_1}{r} + \beta_1 + \frac{\gamma_1 z}{r} & 0 & \frac{\alpha_2}{r} + \beta_2 + \frac{\gamma_2 z}{r} & 0 & \frac{\alpha_3}{r} + \beta_3 + \frac{\gamma_3 z}{r} & 0 \\ 0 & \gamma_1 & 0 & \gamma_2 & 0 & \gamma_3 \\ \gamma_1 & \beta_1 & \gamma_2 & \beta_2 & \gamma_3 & \beta_3 \end{bmatrix}$$

$$\alpha_1 = r_2 z_3 - r_3 z_2 = (8\times 10) - (9\times 7) = 17 \text{ mm}^2$$

$$\alpha_2 = r_3 z_1 - r_1 z_3 = (9\times 7) - (6\times 10) = 3 \text{ mm}^2$$

$$\alpha_3 = r_1 z_2 - r_2 z_1 = (6\times 7) - (8\times 7) = -14 \text{ mm}^2$$

$z_1 = 7$; $z_2 = 7$; $z_3 = 10$

$r_1 = 6$; $r_2 = 8$; $r_3 = 9$

$\beta_1 = z_2 - z_3 = -3\ mm$

$\beta_2 = z_3 - z_1 = 3\ mm$

$\beta_3 = z_1 - z_2 = 0$

$\gamma_1 = r_3 - r_2 = 1\ mm$

$\gamma_2 = r_1 - r_3 = -3\ mm$

$\gamma_3 = r_2 - r_1 = 2\ mm$

$r = \frac{r_1 + r_2 + r_3}{3}$; $z = \frac{z_1 + z_2 + z_3}{3}$

$= \frac{6+8+9}{3}$

$= 23/3$

$r = 7.67\ mm$

$= \frac{7+7+10}{3}$

$= 24/3$

$z = 8\ mm.$

$$\frac{\alpha_1}{r} + \beta_1 + \frac{\gamma_1 z}{r} = \frac{17}{7.67} + (-3) + \frac{1(8)}{7.67} = 0.259 \text{ mm}$$

$$\frac{\alpha_2}{r} + \beta_2 + \frac{\gamma_2 z}{r} = \frac{3}{7.67} + (3) + \frac{-3(8)}{7.67} = 0.262 \text{ mm}$$

$$\frac{\alpha_3}{r} + \beta_3 + \frac{\gamma_3 z}{r} = \frac{-14}{7.67} + (0) + \frac{2(8)}{7.67} = 0.260 \text{ mm}$$

$$\text{Area} = \frac{1}{2} \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} 1 & r_1\ 6 & z_1\ 7 \\ 1 & r_2\ 8 & z_2\ 7 \\ 1 & r_3\ 9 & z_3\ 10 \end{vmatrix} \end{array}$$

$$= \frac{1}{2}\left[1(80-63) - 6(10-7) + 7(9-8)\right]$$

$$= \frac{1}{2}\left[67\right.$$

$$\boxed{A = 3 \text{ mm}^2}$$

$$
B = \frac{1}{2A}\begin{bmatrix}
\beta_1 & 0 & \beta_2 & 0 & \beta_3 & 0 \\
\frac{\alpha_1}{r} + \beta_1 + \frac{\gamma_1 z}{r} & 0 & \frac{\alpha_2}{r} + \beta_2 + \frac{\gamma_2 z}{r} & 0 & \frac{\alpha_3}{r} + \beta_3 + \frac{\gamma_3 z}{r} & 0 \\
0 & \gamma_1 & 0 & \gamma_2 & 0 & \gamma_3 \\
\gamma_1 & \beta_1 & \gamma_2 & \beta_2 & \gamma_3 & \beta_3
\end{bmatrix}
$$

$$
= \frac{1}{2(3)}\begin{bmatrix}
-3 & 0 & 3 & 0 & 0 & 0 \\
0.259 & 0 & 0.262 & 0 & 0.260 & 0 \\
0 & 1 & 0 & -3 & 0 & 2 \\
1 & -3 & -3 & 3 & 2 & 0
\end{bmatrix}
$$

$$B = 0.167 \begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0.259 & 0 & 0.262 & 0 & 0.260 & 0 \\ 0 & 1 & 0 & -3 & 0 & 2 \\ 1 & -3 & -3 & 3 & 2 & 0 \end{bmatrix}_{4 \times 6}$$

$$B^T = 0.167 \begin{bmatrix} -3 & 0.259 & 0 & 1 \\ 0 & 0 & 1 & -3 \\ 3 & 0.262 & 0 & -3 \\ 0 & 0 & -3 & 3 \\ 0 & 0.260 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{bmatrix}_{6 \times 4}$$

Stress strain relationship matrix [D]

$$[D] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 \\ \nu & 1-\nu & \nu & 0 \\ \nu & \nu & 1-\nu & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

$$= \frac{2\times10^5}{(1+0.25)(1-\cancel{0.25}\,0.5)} \begin{bmatrix} 0.75 & 0.25 & 0.25 & 0 \\ 0.25 & 0.75 & 0.25 & 0 \\ 0.25 & 0.25 & 0.75 & 0 \\ 0 & 0 & 0 & 0.25 \end{bmatrix}$$

$$[D] = 8 \times 10^4 \begin{bmatrix} 3 & 1 & 1 & 0 \\ 1 & 3 & 1 & 0 \\ 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$