SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with ‘A+’ Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF AUTOMOBILE ENGINEERING

19AUB303 – Finite Element Methods and Analysis

IV YEAR / VII SEM

UNIT – 3 FORMULATION OF ELEMENT CHARACTERISTIC

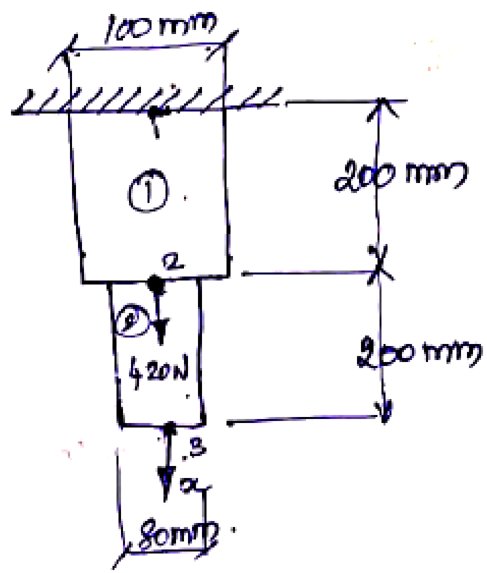
MATRICES AND VECTORS FOR THERMAL PROBLEMS

Topic – 1 – Problems on 1D Plate Elements

1D Bar Element - Problem

A thin steel plate of uniform thickness 25 mm is subjected to a point load of 420 N at mid depth as shown below. The plate is also subjected to self-weight. If Young's modulus, $E = 2 \times 10^5 \ N/mm^2$ & unit weight density, $\rho = 0.8 \times 10^{-4} \ N/mm^3$. Calculate the following (i) displacement at each nodal point

(ii) Stress in each element.



Given Data:-

Thickness, t = 25 mm.

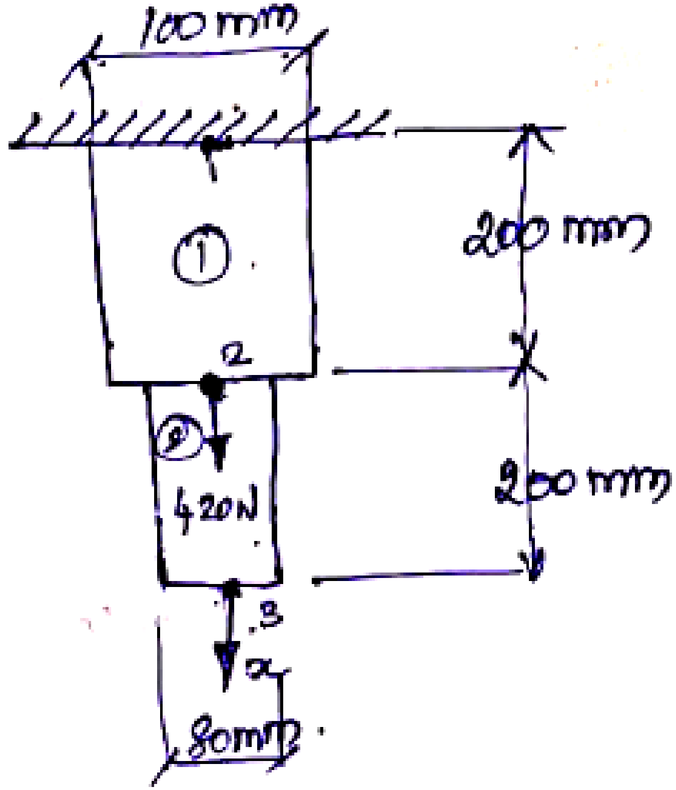
Point Load, P = 420 N.

Young's Modulus, E = 2×10^5 N/mm^2

Unit Weight Density, ρ = 0.8×10^{-4} N/mm^3.

Area, A_1 = 100×25 = 2500 mm^2

Area, A_2 = 80×25 = 2000 mm^2



To Find:-

(i) Displacement at each nodal points, u_1, u_2 & u_3.

(ii) Stress in each element, σ_1 & σ_2

Soln:-

The steel plate is subjected to self weight, we have to find Body Force Vector

$$\text{Body force vector, } \{F\} = \frac{\rho A l}{2}\begin{Bmatrix}1\\1\end{Bmatrix}$$

For element ①,

$$\begin{Bmatrix}F_1\\F_2\end{Bmatrix} = \frac{0.8\times10^{-4}\times 2500\times 200}{2}\begin{Bmatrix}1\\1\end{Bmatrix}$$

$$= 20\begin{Bmatrix}1\\1\end{Bmatrix}$$

$$\begin{Bmatrix}F_1\\F_2\end{Bmatrix} = \begin{Bmatrix}20\\20\end{Bmatrix}$$

For element (2),

$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \frac{0.8\times10^{-4} \times 2000 \times 200}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} 16 \\ 16 \end{Bmatrix},$$

Assembling the body force vector,

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 20+16 \\ 16 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 36 \\ 16 \end{Bmatrix}$$

A point load is acting at mid depth, nodal point 2 $\Rightarrow \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 36 + 420 \\ 16 \end{Bmatrix}$

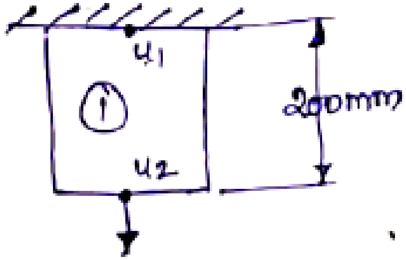
$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 456 \\ 16 \end{Bmatrix}$$

Finite element eqn. for one dimensional plate element;

$$\{F\} = [K]\{u\}$$

$$[K] = \frac{AE}{l}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

For element ①, Nodes (1,2):-



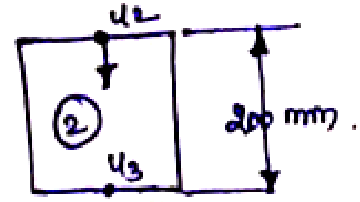
$$\Rightarrow \frac{A_1 E}{l_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$\Rightarrow \frac{2500 \times 2\times10^5}{200}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$\Rightarrow 1\times10^5\begin{bmatrix}25 & -25\\ -25 & 25\end{bmatrix}$$

$$1\times10^5 \begin{bmatrix} \overset{a_{11}}{25} & \overset{a_{12}}{-25} \\ \overset{a_{21}}{-25} & \overset{a_{22}}{25} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \text{ // } \cdots\cdots \rightarrow (A)$$

For element ②, Nodes (2,3):-



$$[k_2] = \frac{A_2 E}{l_2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{2000\times 2\times10^5}{200}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 1\times10^5 \begin{bmatrix} 20 & -20 \\ -20 & 20 \end{bmatrix}.$$

$$\Rightarrow 1\times10^5 \begin{bmatrix} \overset{a_{22}}{20} & \overset{a_{23}}{-20} \\ \overset{a_{32}}{-20} & \overset{a_{33}}{20} \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix}. \cdots\cdots \rightarrow (B)$$

Assembling of Finite Elements, eqn (A) & (B)

$$\Rightarrow 1\times10^5 \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 25 & -25 & 0 \\ a_{21} & a_{22} & a_{23} \\ -25 & 25+20 & -20 \\ a_{31} & a_{32} & a_{33} \\ 0 & -20 & 20 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$$\Rightarrow 1\times10^5 \begin{bmatrix} 25 & -25 & 0 \\ -25 & 45 & -20 \\ 0 & -20 & 20 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

Applying boundary condition; at node 1, the displacement $u_1 = 0$ & substituting the values of F_1, F_2 & F_3.

$$\Rightarrow \quad 1\times10^5 \begin{bmatrix} 25 & -25 & 0 \\ -25 & 45 & -20 \\ 0 & -20 & 20 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 456 \\ 16 \end{Bmatrix}$$

In the above eqn, the first row & first column is neglected because the nodal point ① is fixed.

$$1\times10^5 (45u_2 - \cancel{20u_3}) = 456. \dashrightarrow \text{(C)}$$

$$1\times10^5 (-20u_2 + \cancel{20}u_3) = 16 \dashrightarrow \text{(D)}$$

$$1\times10^5 (25u_2) = 472.$$

$$u_2 = \frac{472}{25\times10^5}$$

$$\boxed{u_2 = 1.888\times10^{-4} \text{ mm}}$$

Sub, the value of $u_2 = 1.888 \times 10^{-4}$ in (c)

$$1\times10^5 (45u_2 - 20u_3) = 456 \rightarrow \text{(c)}$$

$$1\times10^5 \left[(45 \times 1.888 \times 10^{-4}) - 20u_3 \right] = 456.$$

$$1\times10^5 \left[8.496 \times 10^{-3} - 20u_3 \right] = 456.$$

$$849.6 - 20\times10^5 u_3 = 456.$$

$$-\cancel{20\times10^5}\, u_3 = \frac{456 - 849.6}{20\times10^5}$$

$$\cancel{-}u_3 = \cancel{-}1.968 \times 10^{-4}$$

$$\boxed{u_3 = 1.968 \times 10^{-4}\ \text{mm}}$$

(ii) Stress in Each element

For element ①, nodes 1,2 :-

$$\sigma_1 = E\left(\frac{u_2 - u_1}{l_1}\right)$$

$$= 2\times10^5\left[\frac{\left((1.888\times10^{-4}) - 0\right)}{200}\right]$$

$$\boxed{\sigma_1 = 0.1888\ N/mm^2}$$

For element (2), nodes 2,3 :-

$$\sigma_2 = E\left(\frac{u_3 - u_2}{l_2}\right)$$

$$= 2\times10^5\left(\frac{1.968\times10^{-4} - 1.888\times10^{-4}}{200}\right)$$

$$\boxed{\sigma_2 = 0.008\ N/mm^2}$$

Result:-

(i) Displacements at each nodal points

$$u_1 = 0$$

$$u_2 = 1.888 \times 10^{-4} \text{ mm}$$

$$u_3 = 1.968 \times 10^{-4} \text{ mm}.$$

(ii) Stress in each element

$$\sigma_1 = 0.1888 \text{ N/mm}^2 ; \sigma_2 = 0.008 \text{ N/mm}^2$$