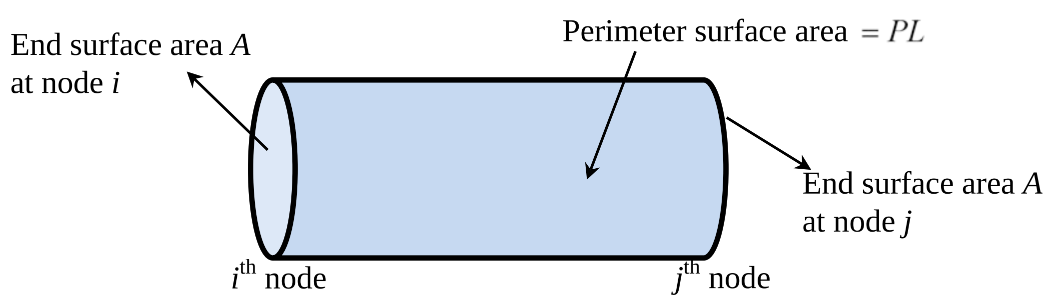
# HEAT TRANSFER THROUGH CONDUCTION AND CONVECTION

| Sl. No. | Material parameter | Symbol | Unit |
|---|---|---|---|
| 1 | **Thermal conductivity of the material**, ***k***<br>Definition: amount of heat conducted between two points at 1 m interval inside a conducting solid when the temperature of the conducting solid is changed by 1 Kelvin or 1 degree centigrade from the ambient temperature. | $k$ | $\frac{W}{mK}$ |
| 2 | **Convection heat transfer coefficient** or<br>**Convection film transfer coefficient, *h***<br>Definition: amount of heat convected from a conducting solid to a conducting fluid over 1 $m^2$ area when the temperature of the conducting solid is changed by 1 Kelvin or 1 degree centigrade from the ambient temperature. | $h$ | $\frac{W}{m^2K}$ |
| 3 | End cross-sectional area of the conducting solid or end area of the prismatic conducting solid | $A$ | $m^2$ |
| 4 | Temperature at any point in the conducting solid | $T$ | $K$ or $^oC$ |
| 5 | Ambient temperature of the surroundings | $T_\infty$ | $K$ or $^oC$ |
| 6 | Perimeter length of the conducting surface | $P$ | $m$ |
| 7 | Length of the conducting solid | $L$ | $m$ |
| 8 | Heat generated per unit volume by a source or sink | $Q$ | $m^3$ |
| | | | |




**Finite element equation for thermal problems**

$$\{f\}_{2\times1} = [k]_{2\times2}\{T\}_{2\times1}$$

$$\text{Watts}=\frac{\text{Watts}}{\text{Kelvin}}\times\ \text{Kelvin} \quad \text{or} \quad \text{Watts}=\frac{\text{Watts}}{\text{degree Celsius}}\times\text{degree Celsius}$$

$\{f\}_{2\times1}$ = Column vector of nodal heat - describes how heat is conducted and/or convected from or to the conducting solid through perimeter surface area and/or end surface area by direct contact or convection and/or internal heat generation (source) / heat dissipation (sink).

$[k]_{2\times2}$ = Thermal stiffness matrix – describes the heat conducted and convected (through various modes as described above) for 1 degree change in temperature.

$\{T\}_{2\times1}$ = Column vector of nodal temperatures.

**STIFFNESS MATRIX TERMS**

1. **Conduction :**

   Amount of heat conducted by the conducting solid $\frac{kA}{L} = \left(\frac{W}{mK}\right)\frac{m^2}{m} = \frac{W}{K}$

2. **Convection through perimeter surface area**

   Amount of heat convected from the conducting solid to the conducting fluid through perimeter surface area $hPL = \left(\frac{W}{m^2K}\right)m \times m = \frac{W}{K}$

3. **Convection through end surface area**

   Amount of heat convected from the conducting solid to the conducting fluid through end surface area – for $i^{th}$ node or $j^{th}$ node only. $hA = \left(\frac{W}{m^2K}\right)m^2 = \frac{W}{K}$

Consider a 1-D 2-noded linear thermal bar element. It has two nodes. At each node, there is only one degree of freedom, namely, temperature, *T*. Hence, there are **two** degrees of freedom per element. And the size of the stiffness matrix is 2×2.

**FORCE MATRIX TERMS**

1. **Perimeter surface area convection**

   Amount of heat through perimeter surface area $hPLT_\infty = \left(\frac{W}{m^2K}\right)\times m \times m \times K = W$

2. **End surface area convection**

   Amount of heat through end surface area $hAT_\infty = \left(\frac{W}{m^2K}\right)\times m^2 \times K = W$

3. **Perimeter surface area heat flux**

   Amount of heat through perimeter surface area $qPL = \left(\frac{W}{m^2}\right)\times m \times m = W$

4. **End surface area heat flux**

   Amount of heat through end surface area $qA = \left(\frac{W}{m^2}\right)\times m^2 = W$

5. **Any internal heat generation or dissipation**

   Amount of heat through end surface area $QAL = \left(\frac{W}{m^3}\right)\times m^2 \times m = W$

**Finite element equation for 1-D thermal conduction and convection problem**

$$[k]_{2\times2} = \frac{KA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{hPL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} + hA\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

| Thermal Stiffness matrix | Axial conduction | Perimeter surface area convection | End surface area convection at $i^{th}$ node |
|---|---|---|---|

or $hA\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

End surface area convection at $j^{th}$ node

$$[f]_{2\times1} = \frac{hPLT_\infty}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} + hAT_\infty\begin{Bmatrix} 1 \\ 0 \end{Bmatrix} + \frac{qPL}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} + qA\begin{Bmatrix} 1 \\ 0 \end{Bmatrix} + \frac{QAL}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

| Column vector of nodal heats | Perimeter surface area convection | End surface area convection at $i^{th}$ node | Perimeter surface area heat flux | End surface area heat flux at $i^{th}$ node | Internal heat generation or dissipation |
|---|---|---|---|---|---|

$hAT_\infty\begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$

End surface area convection at $j^{th}$ node

$qA\begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$

End surface area heat flux at $j^{th}$ node

End surface area heat flux is produced by direct contact of a heat source with the conducting solid. For example, if a heated iron-box is placed on a metal surface, heat is directly transferred from the iron-box to the metal surface through conduction. No convection occurs in this case.