SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution)
Coimbatore-641035.

UNIT-II ORDINARY DIFFERENTIAL EQUATIONS
Linear ODE with constant coefficients

$$
\begin{aligned}
\text { Type } & \text { : } \\
& \text { RHS }=e^{a x} \\
& \text { Replace } D \text { by } a .
\end{aligned}
$$

D. Solve $\left(D^{2}+1\right) y=e^{-x}$

Sols.
The Auxiliary en. is $m^{2}+1=0$

$$
\begin{aligned}
m^{2} & =-1 \\
m & = \pm i
\end{aligned}
$$

$\therefore$ The loots are insaginary.

$$
\begin{aligned}
C F & =e^{O x}[A \cos x+B \sin x] \\
C F & =A \cos x+B \sin x \\
P I & =\frac{1}{D^{2}+1} e^{-x} \\
& =\frac{1}{(-1)^{2}+1} e^{-x} \quad \text { Replace } D \rightarrow a=-1 \\
& =\frac{1}{2} e^{-x} \\
P I & =\frac{e^{-x}}{2}
\end{aligned}
$$

$\therefore$ The soon. is $y=C F+P I$

CS Scanned with

SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution)

Coimbatore-641035.

UNIT-II ORDINARY DIFFERENTIAL EQUATIONS
Linear ODE with constant coefficients

2]. Solve $\left(D^{2}+4 D+4\right) y=11 e^{-2 x}$
sola.
The aureilary eqn. is, $m^{2}+4 m+4=0$
$(m+3)^{2}=0$
$m=-2,-2$
The roots are real and same.
$C F=(A+B x) e^{-2 x}$
$P I=\frac{1}{D^{2}+4 D+4} \| e^{-2 x}$

$$
=11 \frac{1}{(-2)^{2}+4(-2)+4} \bar{e}^{2 x} \quad \quad \begin{aligned}
& \text { Replace } \\
& a \rightarrow a=-2
\end{aligned}
$$

$=11 \frac{1}{4-8+4} e^{-2 x}$
$=11 x \frac{1}{2 D+4} e^{-2 x}$
[If: Dr. $\rightarrow 0$, then
$=11 x \frac{1}{2(-2)+4} e^{-2 x}$ multiply $x$ in the Nr. and differentiate the Dr. wi. to 0$]$
$=11 x^{2} \frac{1}{2} e^{-2 x}$

$$
=\frac{11 x^{2}}{2} e^{-2 x}
$$

$\therefore$ The general soin. is,

$$
\begin{aligned}
y & =C F+P I \\
& =(A+B x) e^{-2 x}+\frac{11 x^{2}}{2} e^{-2 x}
\end{aligned}
$$

3]. Solve

$$
\left(D^{2}-2 x+r\right) y=\cos b x
$$

801n.
Given

$$
\begin{aligned}
\left(D^{2}-2 x+1\right) y & =\frac{e^{x}+e^{-x}}{2} \\
& =\frac{1}{2}\left[e^{x}+e^{-x}\right] \\
& =\frac{e^{x}}{2}+\frac{e^{-x}}{2}
\end{aligned}
$$

SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution)
Coimbatore-641035.

AE

$$
\begin{array}{r}
m^{2}-2 m+1=0 \\
m=1,1
\end{array}
$$

CF

$$
\begin{aligned}
C F & =(A+B x) e^{x} \\
P I_{1} & =\frac{1}{D^{2}-2 D+1} \frac{e^{x}}{2} \\
& =\frac{1}{2} \frac{1}{1^{2}-2(1)+1} e^{x} e^{x} \\
& =\frac{x}{2} \frac{1}{2 D-2} e^{x} \\
& =\frac{x^{2}}{2} \cdot \frac{1}{2} e^{x} \\
P_{I_{1}} & =\frac{x^{2}}{4} e^{x} x \\
P I_{2} & =\frac{1}{D^{2}-2 D+1} \frac{1}{2} \\
& =\frac{1}{2} \frac{1}{e^{2}+2(-1)+1} e^{-x} \\
& =\frac{1}{2} \frac{1}{0}+2+1 \\
P I_{2} & =\frac{1}{8} e^{-x}
\end{aligned}
$$

The general sols. is

$$
\begin{aligned}
& y=C F+P I!+P I_{2} \\
& y=(A+B x) e^{x}+\frac{x^{2}}{4} e^{x}+\frac{1}{8} e^{-x}
\end{aligned}
$$

## CNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore-641035.

UNIT-II ORDINARY DIFFERENTIAL EQUATIONS
Linear ODE with constant coefficients

Type 2:

$$
\begin{aligned}
R H S= & \sin (a x+b) \\
& \text { os } \\
& \cos (a x+b)
\end{aligned}
$$

Replace

$$
D^{2} \rightarrow-a^{2}
$$

ग. Solve $\left(D^{2}+3 x+2\right) y=\sin 3 x$
sol.
CF

$$
\begin{gathered}
m^{2}+3 m+2=0 \\
(m+1)(m+2)=0 \\
m=1,2
\end{gathered}
$$

$$
\because C F=A e^{x}+B e^{2 x}
$$

$$
P I=\frac{1}{D^{2}-3 D+2} \quad \sin 3 x
$$

$$
=\frac{1}{-9-3 D+2} \sin 3 x
$$

$$
=\frac{1}{-3 D-7} \sin 3 x
$$

$$
=\frac{1}{(-3 D-7)} \frac{(-3 D+7)}{(-3 D+7)} \text { sin } 3 x
$$

$$
=\frac{-13 D+7}{9 D^{2}-49} \quad \text { sin } 3 x
$$

$$
=\frac{-3 D+7}{9(-9)-49} \quad \text { sin } 3 x \quad D^{2} \rightarrow-9
$$

$$
=\frac{-3 D+7}{-130} \quad \sin 3 x
$$

$$
\left.=\frac{1}{130} \sum(-30+7) 59 n 3 x\right]
$$

$$
=\frac{-1}{130}[-3 D \sin 3 x+7 \sin 3 x]
$$

CS Scanned with $=\frac{1}{130}[-3(3) \cos 3 x+7 \sin 3 x]$

# CNS COLLEGE OF TECHNOLOGY 

## (An Autonomous Institution)

Coimbatore-641035.

UNIT-II ORDINARY DIFFERENTIAL EQUATIONS
Linear ODE with constant coefficients

$$
\begin{aligned}
& =\frac{1}{r^{2}} \quad 2 e^{x} \quad D \rightarrow a=1 \\
& =x \frac{1}{2 D-3}=e^{x} \\
& =x \frac{1}{2(1)-3} 2 e^{x} \\
& =\frac{x}{-1} 2 e^{x} \\
& =-2 x e^{x} \\
& \therefore \text { The sol. is, } \\
& y=C F+P I_{1}+P I_{2} \\
& =A e^{x}+B e^{2 x}-\frac{1}{20}[6 \sin (2 x+3)+2 \cos (2 x+3)] \\
& -2 x e^{x} \\
& \text { 3. and the PI of } C D^{2}+50+6 x y=\sin 3 x \cos x \\
& \text { Sol. } \\
& \text { Given that }\left(D^{2}+5 D+6\right) y=\sin 3 x \cos x \\
& =\frac{1}{2}[\sin 4 x+\sin 2 x] \\
& \sin A \cos B=\frac{1}{2}[\sin (A+B)+\sin (A-B)] \\
& P I_{1}=\frac{1}{D^{2}+5 D+6} \frac{1}{2} \sin 4 x \\
& =\frac{1}{-16+5 D+6} \quad \frac{1}{2} \sin 4 x \quad D^{2} \rightarrow-a^{2}=-4^{2}=-16 \\
& =\frac{1}{5 D-10} \frac{1}{2} \operatorname{sen} 4 x \\
& =\frac{1}{2} \frac{5 D+10}{25 D^{2}-100} \sin 4 x \\
& =\frac{1}{2} \frac{50 \sin 4 x+10 \sin 4 x}{25(-16)-100} \\
& \text { Scanned with } \\
& \text { Camscanner }
\end{aligned}
$$

$$
\begin{aligned}
& =\frac{1}{2 x(-500)}[20 \cos 4 x+10 \sin 4 x] \\
P I_{1} & =\frac{-1}{+100}[2 \cos 4 x+\sin 4 x] \\
P I_{2} & =\frac{1}{D^{2}+5 D+6} \frac{1}{2} \sin 2 x \\
& =\frac{1}{2} \frac{1}{-4+5 D+6} \sin 2 x \quad D^{2} \rightarrow-a^{2} \\
& =\frac{1}{2} \frac{5 D-2}{25 D^{2}-4} \sin 2 x \\
& =\frac{1}{2} \frac{[5 D \sin 2 x-2 \sin 2 x]}{25(-4)-4} \\
P I_{2} & =-\frac{1}{104}[5 \cos 2 x-\sin 2 x]
\end{aligned}
$$

The sold. is,

$$
\begin{aligned}
y & =F+P I_{1}+P I_{2} \\
& =\frac{-1}{100}[2 \cos 4 x+\sin 4 x]-\frac{1}{104}[5 \cos 2 x-\sin 2 x]
\end{aligned}
$$

SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution) Coimbatore-641035.

SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution)

Coimbatore-641035.

UNIT-II ORDINARY DIFFERENTIAL EQUATIONS
Linear ODE with constant coefficients

$$
\begin{aligned}
& \text { Type 3. RHS }=x^{n} \\
& \text { 1). }(1-D)^{-1}=1+D+D^{2}+D^{3}+\cdots \text {. } \\
& \text { 2). }(1+D)^{-1}=1-D+D^{2}-D^{3}+\cdots \\
& \text { 3). }(1-D)^{-2}=1+2 D+3 D^{2}+A D^{3}+\cdots \cdot \\
& \text { 4). }(1+D)^{-2}=1-2 D+3 D^{2}-4 D^{3}+\cdots \\
& \text { - } \\
& \text { U. Solve }\left(D^{2}+2\right) y=x^{2} \\
& \text { Sol. } \\
& \text { AE } \quad m^{2}+2=0 \\
& m^{2}=-2 \\
& m= \pm \sqrt{2} \text { i } \\
& \alpha \pm i \beta \Rightarrow \alpha=0, \quad \beta=\sqrt{2} \\
& C F=A \cos \sqrt{2} x+B \sin \sqrt{2} x \\
& P I=\frac{1}{D^{2}+2} x^{2} \\
& =\frac{1}{2\left[1+\frac{D^{2}}{2}\right]} x^{2} \\
& =\frac{1}{2}\left[1+\frac{D^{2}}{2}\right]^{-1} x^{2} \\
& =\frac{1}{2}\left[1-\frac{D^{2}}{2}+\frac{D^{4}}{4}-\cdots\right] x^{2} \\
& =\frac{1}{2}\left[1-\frac{D^{2}}{2}\right] x^{2} \quad D^{2} x^{2} \\
& =\frac{1}{2}\left[x^{2}-\frac{D^{2} x^{2}}{2}\right]=\frac{1}{2}\left[x^{2}-\frac{2}{2}\right] \\
& =\frac{1}{2}\left[x^{2}-1\right] \\
& \therefore \text { The sown. } Q s, y=C F+P I \\
& =A \cos \sqrt{2} x+B \sin \sqrt{2} x+\frac{1}{2}\left[x^{2}-1\right]
\end{aligned}
$$

## CNS COLLEGE OF TECHNOLOGY

## (An Autonomous Institution)

Coimbatore-641035.

UNIT-II ORDINARY DIFFERENTIAL EQUATIONS
Linear ODE with constant coefficients

2]. Solve $\left(D^{2}+3 D+2\right) y=x^{2}$
Sols.
$\Delta E$

$$
m^{2}+3 m+2=0
$$

$(m+1)(m+2)=0$.
$m=-1,-2$
$C F=A e^{x}+B e^{-2 x}$
$P I=\frac{1}{D^{2}+3 D+2} x^{2}$

$$
=\frac{1}{2\left[1+\frac{D^{2}+3 D}{2}\right]} x^{2}
$$

$$
=\frac{1}{2}\left[1+\left(\frac{D^{2}+3 D}{2}\right)\right]^{-1} x^{2}
$$

$$
=\frac{1}{2}\left[1-\left(\frac{D^{2}+3 D}{2}\right)+\left(\frac{D^{2}+3 D}{2}\right)^{2}\right] x^{2}
$$

$$
=\frac{1}{2}\left[1-\frac{D^{2}}{2}-\frac{3 D}{2}+\frac{9 D^{2}}{4}\right] x^{2}
$$

$$
=\frac{1}{2}\left[x^{2}-\frac{D^{2} x^{2}}{2}-\frac{3 D}{2} x^{2}+\frac{9 D^{2}}{4} x^{2}\right] \quad D^{\prime}=2 x
$$

$$
=\frac{1}{2}\left[x^{2}-\frac{2}{2}-\frac{3(2 x)}{2}+9(2)\right] \quad D^{\prime \prime}=2
$$

$$
=\frac{1}{2}\left[x^{2}-1-3 x+9 / 2\right]
$$

$$
P I=\frac{1}{2}\left[x^{2}-3 x+\frac{7}{2}\right]
$$

$$
\text { The sorn. is, } y=C F+P I
$$

$$
\text { CS } \begin{aligned}
& y=A e^{x}+B e^{-2 x}+\frac{1}{2}\left[x^{2}-3 x+\frac{7}{2}\right] \\
& \text { Camscanner }
\end{aligned}
$$

## CNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore-641035.

UNIT-II ORDINARY DIFFERENTIAL EQUATIONS
Linear ODE with constant coefficients

Type -4

$$
\text { RHS }=e^{a x} \dot{\phi}(x) \text { where } \phi(x)=\begin{array}{r}
\sin b x \text { or } \\
\cos b x \text { or }
\end{array}
$$

Replace $D \rightarrow D+a$ $x^{m}$
I. Solve $\left(D^{2}-4 D+3\right) y=e^{x} \cos 2 x$

Sols.
CS

$$
\begin{aligned}
& m^{2}-4 m+3=0 \\
& m=1,3 \\
& C F=A e^{x}+B e^{3 x} \\
& P I=\frac{1}{D^{2}-4 D+3} e^{x} \cos 2 x \\
& =e^{x} \frac{1}{(D+1)^{2}-4(D+1)+3} \cos 2 x \quad D \rightarrow D+a=D+1 \\
& =e^{x} \frac{1}{D^{2}+1+2 D-4 D-4+3} \cos 2 x \\
& =e^{x} \frac{1}{D^{2}-2 D} \cos 2 x \\
& =2=e^{x} \frac{1}{-4-2 D} \cos 2 x \rightarrow D^{2} \rightarrow-a^{2}=-2^{2}=-4 \\
& =e^{x} \frac{-2 D+4}{4 D^{2}-16} \cos 2 x \\
& =e^{x} \frac{-2 D \cos 2 x+4 \cos 2 x}{4(-4)-16} \\
& =\frac{e^{x}}{-32}\left[4 \sin 2 x+4 \cos ^{\prime} 2 x\right] \text {. } \\
& =-\frac{e^{x}}{8}[\sin 2 x+\cos 2 x] \\
& \therefore \text { The Sown. is } y=C F+P I \\
& y=A e^{x}+B e^{3 x}-\frac{e^{x}}{8}[\sin 2 x+\cos 2 x] \\
& \text { CamScanner }
\end{aligned}
$$

SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution) Coimbatore-641035.

## CNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore-641035.

UNIT-II ORDINARY DIFFERENTIAL EQUATIONS
Linear ODE with constant coefficients

$$
\begin{aligned}
& \text { 2]. Fixed the PI of } \frac{d^{2} y}{d x^{2}}+4 \frac{d y}{d x}+4 y=x e^{2} x \\
& \text { Sown. } \\
& \text { Given that }\left(D^{2}+4 D+4\right) y=x e^{-2 x} \\
& P I=\frac{1}{D^{2}+4 D+4} e^{-2 x} x \\
& =e^{-2 x} \frac{1}{(D-2)^{2}+4(D-2)+4} \quad x \quad D \rightarrow D+a=D-\cdot 2 \\
& =\bar{e}^{2 x} \frac{1}{D^{2}+4-4 D+4 D-8+4} x \\
& =e^{-2 x} \frac{1}{D^{2}} x \\
& P I=\frac{e^{2 x} x^{3}}{6} \\
& \frac{1}{D} x=\int x=\frac{x^{2}}{2} \\
& \frac{1}{D^{2}}=\frac{x^{3}}{6}
\end{aligned}
$$

HO
ग. Solve $\left(D^{2}-4 D-5\right) y=x e^{x}$
2J. Solve $\left(D^{2}+4 D+4\right) y=e^{2 x} x^{2}$
3]. $\left(D^{2}+4 D+4\right) y=e^{2 x} \sin x$

Type -5
case 1: $\quad$ HS $=x \phi(x)$ where $\phi(x)=\sin a x$ $P I=x \frac{1}{f(D)} \phi(x)-\frac{f^{\prime}(D)}{[F(D)]^{2}} \phi(x) \quad \cos a x$ case 2:


## SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore-641035.

UNIT-II ORDINARY DIFFERENTIAL EQUATIONS
Linear ODE with constant coefficients

ग. Solve $\left(D^{2}+4\right) y=x \sin x$
Soln.

$$
\begin{aligned}
& m^{2}+4=0 \\
& m^{2}=-4 \\
& m= \pm 2 i \\
& \alpha=0, \quad \beta=2 \\
& C F=A \cos 2 x+B \sin D x \\
& P I=\frac{1}{D^{2}+4} x \sin x \\
& =x \frac{1}{D^{2}+4} \sin x-\frac{2 D}{\left(D^{2}+4\right)^{2}} \sin x . \\
& =x \frac{1}{-1+4} \sin x-\frac{4 \cos x}{(-1+4)^{2}} \quad D^{2} \rightarrow-a^{2}=-1^{2}=-1 \\
& =\frac{x \sin x}{3}-\frac{4 \cos x}{9} \\
& \therefore \text { The soln. Is } y=C F+P I \\
& y=A \cos 2 x+B \sin 2 x+\frac{x \sin x}{3}- \\
& \frac{4 \cos x}{9} \\
& \text { 2]. Solve }\left(D^{2}-2 D+1\right) y=x e^{x} \sin x \\
& \text { Soln. } \\
& m^{2}-2 m+1=0 \\
& \infty=1 \text {, } 1 \\
& C F=(A+B x) e^{x} \\
& P I=\frac{1}{D^{2}-2 D+1} e^{x} x \sin x \\
& =e^{x} \frac{1}{(D+p)^{2}-2(D+1)+5} x \sin x \quad D \rightarrow D+a \\
& \text { CS Scanned with } e^{x} \frac{1}{D^{2}+1+2 D-2 D-2+1} x \sin x
\end{aligned}
$$

$$
\begin{aligned}
& =e^{x} \frac{1}{D^{2}} x \sin x \\
& =e^{x}\left[x \frac{1}{D^{2}} \sin x-\frac{2 D}{D^{4}} \sin x\right] \\
& =e^{x}\left[x \frac{1}{-1} \sin x-\frac{2 \cos x}{(-1)^{2}}\right] \\
P I & =-x e^{x} \sin x-2 e^{x} \cos x
\end{aligned}
$$

$$
\begin{aligned}
& \text { The soln. is, } \\
& y=C F+P I \\
&=(A+B x) e^{x}-x e^{x} \sin x-2 e^{x} \cos x
\end{aligned}
$$

CS
Scanned with CamScanner

