

SNS COLLEGE OF TECHNOLOGY



(An Autonomous Institution) Coimbatore-641035.

UNIT-II ORDINARY DIFFERENTIAL EQUATIONS Legendre's Linear Differential Equation Legendre 's linear differential Equation An eqn. of the form  $\frac{(ax+b)^{n}}{dx^{n}} + a_{1}(ax+b)^{n-1} \frac{d^{n-1}y}{dx^{n-1}} + a^{2}(ax+b)^{n-2} \frac{d^{n-2}y}{dx^{n-2}}$ +.... +  $a_{n-1}(ax+b)\frac{dy}{dx}$  +  $a_n y = \Theta(x) \rightarrow (i)$ Take ax+B = p2  $z = \log(ax+b)$ (ax+b)D = aD' $(ax+b)^{d} D^{d} = a^{d} D' (D'-1)$  $(a_{2}+b_{2}^{3})^{3} = a^{3} D'(D'-1)(D'-2)$  and so on. J. Transform the equation to constant wether sents ogn.  $(2x+3)^{2}y'' - (2x+3)y' + 2y = 6x$ Soln. Given [Qx+3]2 p2- (2x+3) D+2]y= 6x Take  $ax+3 = e^{z} \Rightarrow ax = e^{z} = 3$   $z = log(ax+3) \qquad b = e^{z} = 3$ (2x+3)D = 2D' $(22+3)^2 D^2 = 4 D' (D'-1)$  $(1) \Rightarrow \left[4D'(D'-1) - 2D' + 2\right] y = 6\left[\frac{e^{x} - 3}{2}\right]$  $[4D'^{2} + D' - 2D' + 2] y = 3[e^{2} - 3]$  $[4D'^{2} 6D' + 2]y = 3e^{2} - 9$  which is a linear eqn. with constant coeffectents. 2]. Solve  $(x+2)^{2} \frac{d^{2}y}{dx^{2}} - (x+2) \frac{dy}{dx} + y = 3x + 4$ Scanned CamScanner



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UNIT-II ORDINARY DIFFERENTIAL EQUATIONS Legendre's Linear Differential Equation Sola Given  $\int (2(+2)^2 D^2 - (2(+2) D + 1] y = 32(+4 - -2(1))$ Take  $x+2=e^{\chi} \Rightarrow \chi=e^{\chi}-2$  $x = \log(x+2)$ (2+2)D = D' $(2+2)^2 D^2 = D' (D'-1)$  $(1) \Rightarrow \int D'(D'-1) - D' + i J = 3 [e^{2} - 2] + 4$  $\begin{bmatrix} D'(D'-1) - \nu + y - y \\ TD'^{2} - D' - D' + i \end{bmatrix} y = 3e^{7} - 6 + 4$  $[D^2 - 2D' + 1]y = 3e^7 - 2$ CE  $m^2 - am + r = 0$ 1 . t . . . (m-1) (m-1) = 0ham = L, ) - h alteration  $CF = (A + Bz) p^{Z}$  $PT_{2} = \frac{1}{D^{2} - 2D' + 1} = \frac{3e^{2} + \frac{1}{D^{2} - 2D' + 1}} = \frac{1}{D^{2} - 2D' + 1} = \frac{1}{D^$  $D^{2} = 2D' + 1$   $= \frac{1}{1 - 2 + 1} = 3e^{2} - 2 \frac{1}{1}e^{02}$   $D' \Rightarrow a = 1$  a = 0 $= \chi \frac{1}{2H-2} = 3e^{\chi} - 2$  $= \chi \frac{1}{2(1)-2} = 3e^{2} - 2$ = z2 - 3ez - 2  $PT = \frac{37^2 e^7}{9} - 2$ The Soln 95, y=CF+PI =(A+Bz)er + 32rer - 2 Scanned with=  $[A + B \log (x+2)](x+2) + \frac{3[\log (x+2)]^2(x+2)}{2}$ CamScanne