Unit - I

Differential Equation:

An eqn. involving differential coefficients or derivatives is called differential eqn.

Ordinary differential eqn.

A differential eqn. which depends on only one independent variable is called ordinary differential eqn.

Order and degree:

* The order of the highest derivative occurring in the gvn. eqn is called the order of a differential eqn.

* The degree of the highest derivative occurring in the gvn. eqn. is called the degree of a differential eqn.

Second order linear ODE with constant coefficients:

The general linear ODE with constant coefficients is of the form

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} + \dots + a_n y = f(x) \quad \to (1)$$

where $a_0, a_1, \dots a_n$ are constants and $f(x)$ is a function of x.

When $f(x) = 0$ in (1) is called homogeneous ODE & If $f(x) \neq 0$ in (1) is called non homogeneous ODE.

This eqn. can be written as,

$$[a_0 D^n + a_1 D^{n-1} + a_2 D^{n-2} + \dots + a_n) y = f(x).$$

where $D = \frac{d}{dx}$

Solution = CF + PI

= Complementary function + Particular integ

To find CF :

Roots	CF
i). Roots are real & different $m_1 \neq m_2 \neq \dots$	$A e^{m_1 x} + B e^{m_2 x}$
ii). Roots are real & same $m_1 = m_2 = m$	$(A + Bx) e^{mx}$
iii). Roots are imaginary. (or complex) $m = \alpha \pm i\beta$	$e^{\alpha x}[A \cos \beta x + B \sin \beta x]$

To find PI :

$$PI = \frac{1}{f(D)} f(x)$$

RHS = 0

1). Solve $(D^2 - 5D + 6) = 0$

Soln.

The auxiliary eqn. is

$$(m^2 - 5m + 6 = 0$$

$$(m-3)(m-2) = 0$$

$$m = 2, 3$$

∴ The roots are real and different.

$$CF = Ae^{2x} + Be^{3x}$$

$$\therefore\ y = CF = Ae^{2x} + Be^{3x}$$

2]. Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$

Soln.

$$(D^2 - 6D + 9)y = 0$$

The Auxiliary eqn is

$$m^2 - 6m + 9 = 0$$

$$(m-3)^2 = 0$$

$$m = 3, 3$$

The roots are real and same.

$$CF = (A + Bx)e^{3x}$$

$$\therefore\ y = CF = (A + Bx)e^{3x}$$

3]. Solve $(D^2+1)^2 y = 0$

Soln.

The Auxiliary eqn. is $(m^2+1)^2 = 0$

Taking square root on both sides

$$m^2 + 1 = 0$$

$$m^2 = -1$$

$$m = \pm i$$

The roots are imaginary

Here $\alpha = 0,\ \beta = 1$

$$\therefore\ CF = e^0 (A\cos x + B\sin x)$$

$$= A\cos x + B\sin x$$

$$\therefore\ y = CF = A\cos x + B\sin x$$

4]. Solve $(D^4 - 1)y = 0$

Soln. The auxiliary eqn. is $m^4 - 1 = 0$

$$(m^2)^2 - 1^2 = 0$$

$$(m^2+1)(m^2-1) = 0$$

$m^2+1=0$	$m^2-1=0$
$m^2=-1$	$m^2=1$
$m=\pm i$	$m=\pm 1$

$$\therefore CF = Ae^{x} + Be^{-x} + C\cos x + D\sin x.$$