SNS COLLEGE OF TECHNOLOGY

Coimbatore-26
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DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT212 ⸗ CONTROL SYSTEMS

II YEAR/ IV SEMESTER

UNIT II ⸗ TIME RESPONSE ANALYSIS

TOPIC 5- STEADY STATE ERRORS

OUTLINE

- REVIEW ABOUT PREVIOUS CLASS
- STEADY STATE ERROR
- CLASSIFICATION OF CONTROL SYSTEMS
- STEADY STATE ERROR OF UNITY FEEDBACK SYSTEMS
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- STATIC ACCELERATION ERROR CONSTANT (K_A)
- EXAMPLE
- SUMMARY

STEADY STATE ERROR

- If the output of a control system at steady state does not exactly match with the input, the system is said to have steady state error

- Any physical control system inherently suffers steady-state error in response to certain types of inputs.

- A system may have no steady-state error to a step input, but the same system may exhibit nonzero steady-state error to a ramp input.

CLASSIFICATION OF CONTROL SYSTEMS

- Control systems may be classified according to their ability to follow
  - Step inputs,
  - Ramp inputs,
  - Parabolic inputs, and so on.

- The magnitudes of the steady-state errors due to these individual inputs are indicative of the goodness of the system.

CLASSIFICATION OF CONTROL SYSTEMS

- Consider the unity-feedback control system with the following open-loop transfer function

$$G(s) = \frac{K(T_a s + 1)(T_b s + 1)\cdots(T_m s + 1)}{s^N(T_1 s + 1)(T_2 s + 1)\cdots(T_p s + 1)}$$

- It involves the term s^N in the denominator, representing N poles at the origin.

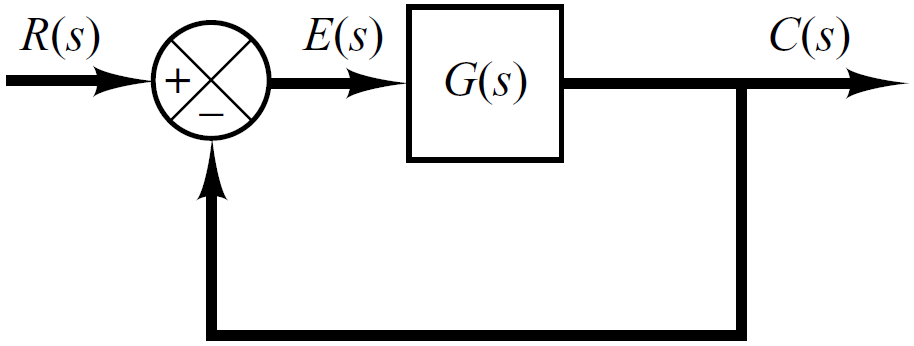
- A system is called type 0, type 1, type 2, ... ,
  if N=0, N=1, N=2, ... , respectively.

CLASSIFICATION OF CONTROL SYSTEMS

- As the type number is increased, accuracy is improved.
- However, increasing the type number aggravates the stability problem.
- A compromise between steady-state accuracy and relative stability is always necessary.

STEADY STATE ERROR OF UNITY FEEDBACK SYSTEMS

- Consider the system shown in following figure.



- The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)} \qquad G(s) = \frac{K(T_a s + 1)(T_b s + 1)\cdots(T_m s + 1)}{s^N(T_1 s + 1)(T_2 s + 1)\cdots(T_p s + 1)}$$

STEADY STATE ERROR OF UNITY FEEDBACK SYSTEMS

- Steady state error is defined as the error between the input signal and the output signal when $t \to \infty$.

- The transfer function between the error signal E(s) and the input signal R(s) is $$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)}$$
- The final-value theorem provides a convenient way to find the steady-state performance of a stable system.
- Since E(s) is $$E(s) = \frac{1}{1 + G(s)} R(s)$$
- The steady state error is

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{1 + G(s)}$$

STATIC ERROR CONSTANTS

- The static error constants are figures of merit of control systems. The higher the constants, the smaller the steady-state error.

- In a given system, the output may be the position, velocity, pressure, temperature, or the like.

- Therefore, in what follows, we shall call the output "position," the rate of change of the output "velocity," and so on.

- This means that in a temperature control system "position" represents the output temperature, "velocity" represents the rate of change of the output temperature, and so on.

STATIC POSITION ERROR CONSTANT (K_P)

- The steady-state error of the system for a unit-step input is

$$e_{ss} = \lim_{s \to 0} \frac{s}{1 + G(s)} \frac{1}{s}$$

$$= \frac{1}{1 + G(0)}$$

- The static position error constant K_p is defined by

$$K_p = \lim_{s \to 0} G(s) = G(0)$$

- Thus, the steady-state error in terms of the static position error constant K_p is given by

$$e_{ss} = \frac{1}{1 + K_p}$$

STATIC POSITION ERROR CONSTANT (K_P)

- For a Type 0 system

$$K_p = \lim_{s \to 0} \frac{K(T_a s + 1)(T_b s + 1)\cdots}{(T_1 s + 1)(T_2 s + 1)\cdots} = K$$

- For Type 1 or higher order systems

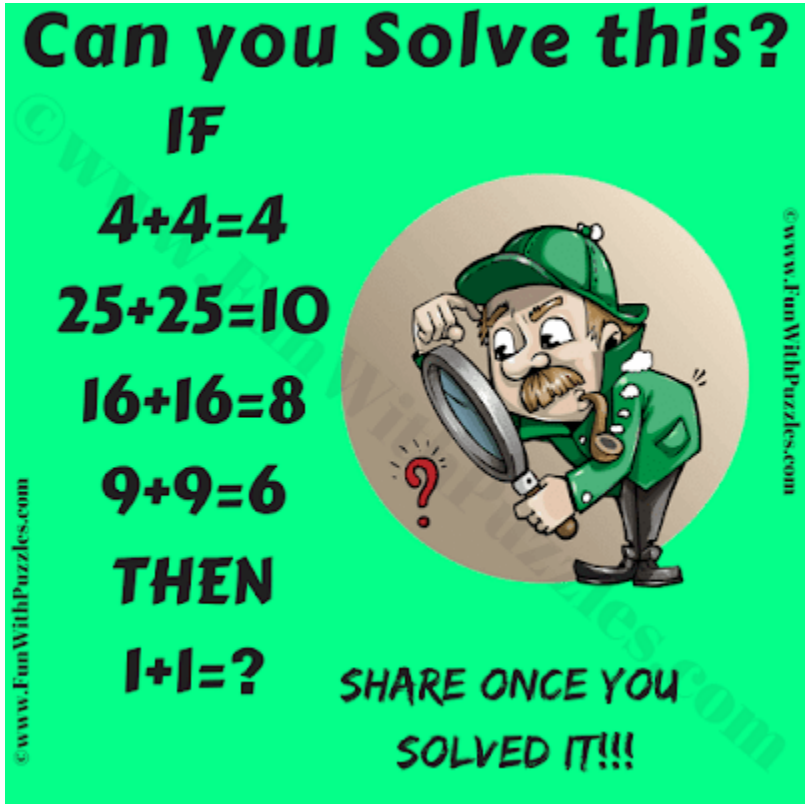
$$K_p = \lim_{s \to 0} \frac{K(T_a s + 1)(T_b s + 1)\cdots}{s^N(T_1 s + 1)(T_2 s + 1)\cdots} = \infty, \qquad \text{for } N \geq 1$$

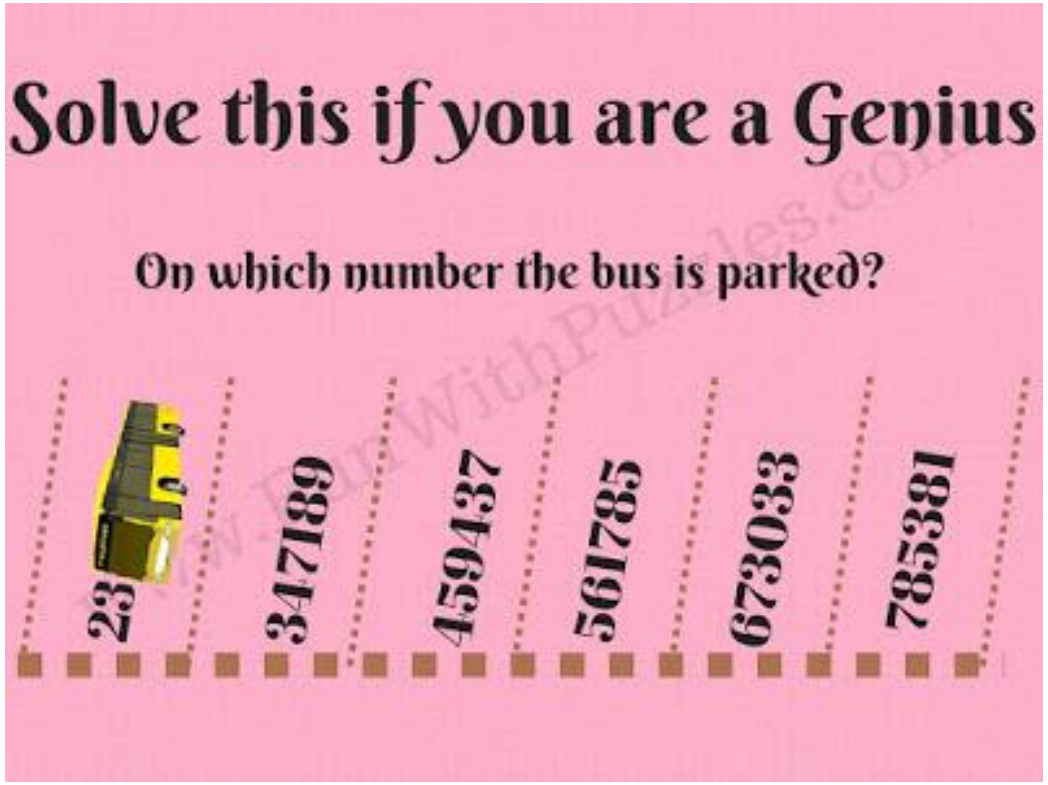
- For a unit step input the steady state error e_{ss} is

$$e_{ss} = \frac{1}{1 + K}, \qquad \text{for type 0 systems}$$

$$e_{ss} = 0, \qquad \text{for type 1 or higher systems}$$

ACTIVITY-PUZZLES





STATIC VELOCITY ERROR CONSTANT (K_V)

- The steady-state error of the system for a unit-ramp input is

$$e_{ss} = \lim_{s \to 0} \frac{s}{1 + G(s)} \frac{1}{s^2}$$

$$= \lim_{s \to 0} \frac{1}{sG(s)}$$

- The static velocity error constant K_v is defined by

$$K_v = \lim_{s \to 0} sG(s)$$

- Thus, the steady-state error in terms of the static velocity error constant K_v is given by

$$e_{ss} = \frac{1}{K_v}$$

STATIC VELOCITY ERROR CONSTANT (K_V)

- For a Type 0 system

$$K_v = \lim_{s \to 0} \frac{sK(T_a s + 1)(T_b s + 1)\cdots}{(T_1 s + 1)(T_2 s + 1)\cdots} = 0$$

- For Type 1 systems

$$K_v = \lim_{s \to 0} \frac{sK(T_a s + 1)(T_b s + 1)\cdots}{s(T_1 s + 1)(T_2 s + 1)\cdots} = K$$

- For type 2 or higher order systems

$$K_v = \lim_{s \to 0} \frac{sK(T_a s + 1)(T_b s + 1)\cdots}{s^N(T_1 s + 1)(T_2 s + 1)\cdots} = \infty, \qquad \text{for } N \geq 2$$

STATIC VELOCITY ERROR CONSTANT (K_V)

- For a ramp input the steady state error e_{ss} is

$$e_{ss} = \frac{1}{K_v} = \infty, \qquad \text{for type 0 systems}$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{K}, \qquad \text{for type 1 systems}$$

$$e_{ss} = \frac{1}{K_v} = 0, \qquad \text{for type 2 or higher systems}$$

STATIC ACCELERATION ERROR CONSTANT (K_A)

- The steady-state error of the system for parabolic input is

$$e_{ss} = \lim_{s \to 0} \frac{s}{1 + G(s)} \frac{1}{s^3}$$

$$= \frac{1}{\lim_{s \to 0} s^2 G(s)}$$

- The static acceleration error constant K_a is defined by

$$K_a = \lim_{s \to 0} s^2 G(s)$$

- Thus, the steady-state error in terms of the static acceleration error constant K_a is given by

$$e_{ss} = \frac{1}{K_a}$$