1) Design a FIR low pass filter with cutoff frequency of 1 KHz and Sampling freq of 4 KHz with 11 samples using Fourier Series Method.

Soln : $F_c = 1\ KHz \qquad F_s = 4\ KHz$

$$\therefore\ w_c = \Omega_c T = \frac{\Omega_c}{F_s} = \frac{2\pi f_c}{F_s} = \frac{2\pi \times 1 \times 10^3}{4 \times 10^3} = 0.5\ \text{rad/sam}$$

The desired frequency response $H_d(e^{jw})$ of lowpass filter is

$$H_d(e^{jw}) = \begin{cases} 1 & \text{for } -w_c \le w \le w_c \\ 0 & \text{for } -\pi \le w \le -w_c \text{ and } w_c \le w \le \pi \end{cases}$$

The desired impulse response $h_d(n)$ of the lowpass filter is

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{jw})\, e^{jwn}\, dw$$

$$= \frac{1}{2\pi} \int_{-w_c}^{w_c} 1\, e^{jwn}\, dw \Rightarrow \frac{1}{2\pi} \left[\frac{e^{jwn}}{jn} \right]_{-w_c}^{w_c}$$

$$= \frac{1}{2\pi} \left[\frac{e^{jw_c n}}{jn} - \frac{e^{-jw_c n}}{jn} \right]$$

$$= \frac{1}{\pi n} \left[\frac{e^{jw_c n} - e^{-jw_c n}}{2j} \right] \qquad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$= \frac{1}{\pi n} \sin w_c n \quad \text{for all } n, \text{ except } n=0$$

when $n=0$; $h_d(n) = h_d(0) = \lim_{n \to 0} \frac{\sin w_c n}{\pi n} = \frac{1}{\pi} \lim_{n \to 0} \frac{\sin w_c n}{n}$

$$= \frac{w_c}{\pi}$$

The impulse response $h(n)$ of FIR filter is obtained by truncating $h_d(n)$ to 11 samples

Using L' hospital rule $\underset{\theta \to 0}{Lt} \frac{\sin A\theta}{\theta} = A$

$$\therefore h(n) = h_d(n) = \frac{\sin \omega_c n}{n\pi} \quad \text{for } n = -\frac{N-1}{2} \text{ to } \frac{N-1}{2}$$

$$= \frac{\omega_c}{\pi} \quad \text{for } n = 0$$

Here $N = 11$, $\therefore \frac{N-1}{2} = \frac{11-1}{2} = 5$

Hence calculate $h(n)$ for $n = -5$ to $+5$

Since, the impulse response $h(n)$ satisfies the symmetry condition $h(-n) = h(n)$ $\therefore$ calculate $h(n)$ for $n = 0$ to 5.

when $n=0$; $h(0) = \frac{\omega_c}{\pi} = 0.5$

when $n=1$; $h(1) = \frac{\sin(0.5\pi \times 1)}{\pi \times 1} = 0.3183$

when $n=2$; $h(2) = \frac{\sin(0.5\pi \times 2)}{\pi \times 2} = 0$

when $n=3$; $h(3) = \frac{\sin(0.5\pi \times 3)}{\pi \times 3} = -0.1061$

when $n=4$; $h(4) = \frac{\sin(0.5\pi \times 4)}{\pi \times 4} = 0$

when $n=5$; $h(5) = \frac{\sin(0.5\pi \times 5)}{\pi \times 5} = 0.0637$

when $n=-1$; $h(-1) = h(1) = 0.3183$

when $n=-2$; $h(-2) = h(2) = 0$

when $n=-3$; $h(-3) = h(3) = -0.1061$

when $n=-4$; $h(-4) = h(4) = 0$

when $n=-5$; $h(-5) = h(5) = 0.0637$

The transfer function $H(z)$ of the digital lowpass filter is given by

$$H(z) = z^{-\frac{N-1}{2}} \; z\{h(n)\} = z^{-\frac{N-1}{2}} \left[\sum_{n=-\frac{N-1}{2}}^{\frac{N-1}{2}} h(n)\, z^{-n} \right] = z^{-5} \sum_{n=-5}^{5} h(n)\, z^{-n}$$

$$= z^{-5} \Big[h(-5) z^{5} + h(-4) z^{4} + h(-3) z^{3} + h(-2) z^{2} + h(-1) z + h(0) z^{0} + h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + h(4) z^{-4} + h(5) z^{-5} \Big]$$

Using symmetry condition $h(-n) = h(n)$

$$= z^{-5} \Big[h(5) z^{5} + h(4) z^{4} + h(3) z^{3} + h(2) z^{2} + h(1) z + h(0) z^{0} + h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + h(4) z^{-4} + h(5) z^{-5} \Big]$$

$$= z^{-5} \Big[h(0) + h(1)\left[z + z^{-1}\right] + h(2)\left[z^{2} + z^{-2}\right] + h(3)\left[z^{3} + z^{-3}\right] + h(4)\left[z^{4} + z^{-4}\right] + h(5)\left[z^{5} + z^{-5}\right] \Big]$$

$$= h(0) z^{-5} + h(1)\left[z^{-4} + z^{-6}\right] + h(2)\left[z^{-3} + z^{-7}\right] + h(3)\left[z^{-2} + z^{-8}\right] + h(4)\left[z^{-1} + z^{-9}\right] + h(5)\left[z^{0} + z^{-10}\right]$$

$$H(z) = 0.5 z^{-5} + 0.3183\left[z^{-4} + z^{-6}\right] - 0.1061\left[z^{-2} + z^{-8}\right] + 0.0637\left[1 + z^{-10}\right]$$

structure :-

$$H(z) = \frac{Y(z)}{X(z)}$$

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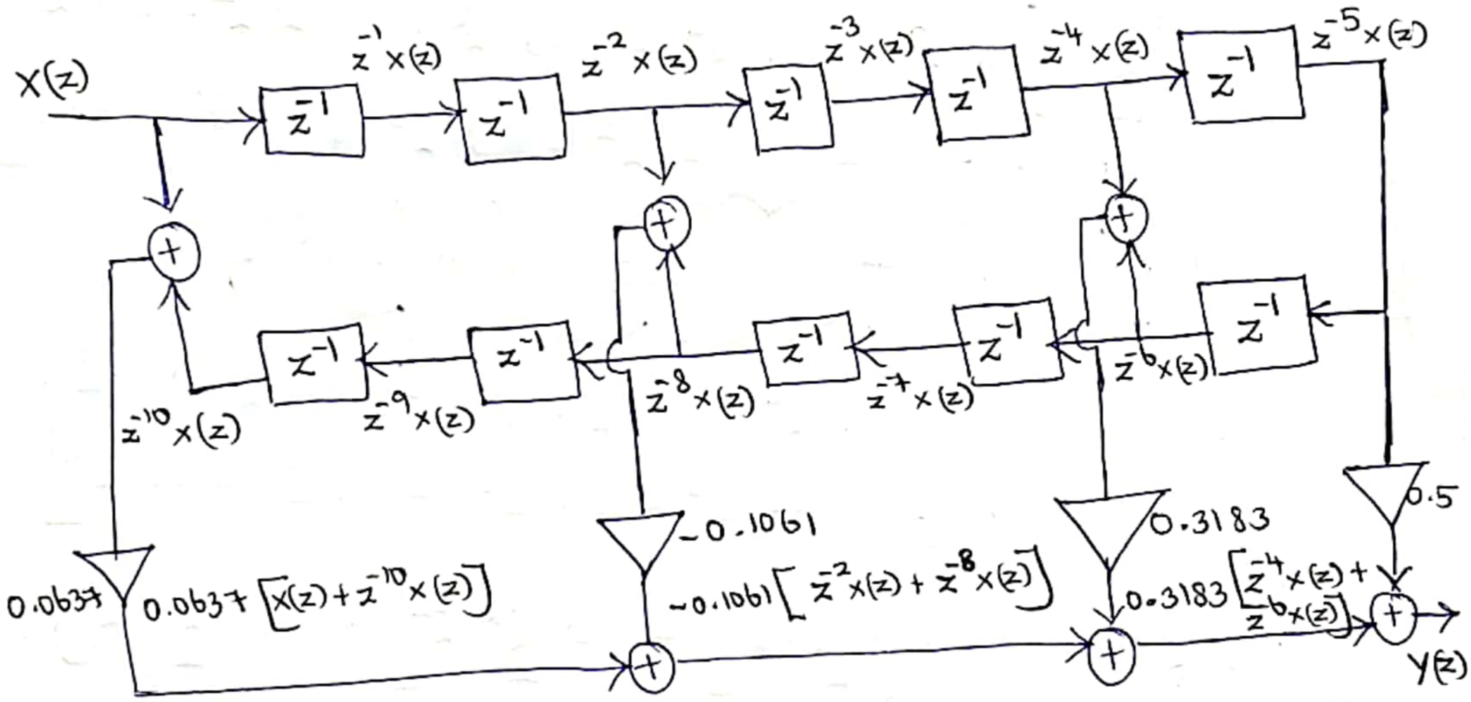
$$H(z) = \frac{Y(z)}{X(z)} = 0.5z^{-5} + 0.3183\left[z^{-4} + z^{-6}\right] - 0.1061\left[z^{-2} + z^{-8}\right] + 0.0637\left[1 + z^{-10}\right]$$

$$\therefore\; Y(z) = 0.5z^{-5}X(z) + 0.3183\left[z^{-4}X(z) + z^{-6}X(z)\right] - 0.1061\left[z^{-2}X(z) + z^{-8}X(z)\right] + 0.0637\left[X(z) + z^{-10}X(z)\right]$$



Linear phase structure of FIR Low pass filter

Freq Response :-

$$A(\omega) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} 2h(n)\cos\omega n$$

$$= h(0) + \sum_{n=1}^{5} 2h(n)\cos\omega n$$

$$= h(0) + 2h(1)\cos\omega + 2h(2)\cos 2\omega + 2h(3)\cos 3\omega + 2h(4)\cos 4\omega + 2h(5)\cos 5\omega$$

$$= 0.5 + 2\times 0.3183\cos\omega + 2\times 0\times\cos 2\omega + 2\times -0.1061\cos 3\omega + 2\times 0\cos 4\omega + 2\times 0.0637\cos 5\omega$$

$$A(\omega) = 0.5 + 0.6366\cos\omega - 0.2122\cos 3\omega + 0.1274\cos 5\omega$$