SNS COLLEGE OF TECHNOLOGY

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Coimbatore-35

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DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECB212 – DIGITAL SIGNAL PROCESSING

II YEAR/ IV SEMESTER

UNIT 3 – FIR FILTER DESIGN

TOPIC – Linear Phase FIR Filter

FREQUENCY RESPONSE OF LINEAR PHASE FIR FILTERS

- Depending on the value of N (Odd or Even) and the type of symmetry of the filter impulse response sequence (Symmetric or Antisymmetric) there are six possible types of linear phase FIR filters
- Symmetric impulse response and N is odd with centre of symmetry at (N-1)/2
- Symmetric impulse response and N is even with centre of symmetry at (N-1)/2
- Antisymmetric impulse response & N is odd with centre of antisymmetry at (N-1)/2
- Antisymmetric impulse response & N is even with centre of antisymmetry at (N-1)/2
- Symmetric impulse response and N is odd with centre of symmetry at n=0
- Antisymmetric impulse response & N is odd with centre of antisymmetry at n=0

FREQUENCY RESPONSE CHARACTERISTICS OF LINEAR PHASE FIR FILTERS

Case	Impulse Response h(n)	N	Symmetry Condition	A(ω) [Amplitude Function $H(e^{j\omega})$]
1	Symmetric	Odd	h (N-1-n) = h(n)	Symmetric
2	Symmetric	Even	h (N-1-n) = h(n)	Antisymmetric
3	Antisymmetric	Odd	h (N-1-n) = - h(n)	Antisymmetric
4	Antisymmetric	Even	h (N-1-n) = - h(n)	Symmetric
5	Symmetric	Odd	h(-n) = h(n)	Symmetric
6	Antisymmetric	Odd	h(-n) = -h(n)	Antisymmetric

SUMMARY OF A(ω) FOR LINEAR PHASE FIR FILTERS

Case	Impulse Response h(n)	N	Symmetry Condition	Magnitude Function $\lvert H(e^{j\omega})\rvert = \lvert A(\omega)\rvert$
1	Symmetric	Odd	h (N-1-n) = h(n)	$\left\lvert h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} 2\,h\left(\frac{N-1}{2} - n\right) \cos \omega n \right\rvert$
2	Symmetric	Even	h (N-1-n) = h(n)	$\left\lvert \sum_{n=1}^{\frac{N}{2}} 2\,h\left(\frac{N}{2} - n\right) \cos\left(\omega\left(n - \frac{1}{2}\right)\right) \right\rvert$
3	Antisymmetric	Odd	h (N-1-n) = - h(n)	$\left\lvert \sum_{n=1}^{\frac{N-1}{2}} 2\,h\left(\frac{N-1}{2} - n\right) \sin \omega n \right\rvert$

SUMMARY OF A(ω) FOR LINEAR PHASE FIR FILTERS

Case	Impulse Response h(n)	N	Symmetry Condition	Magnitude Function $\lvert H(e^{j\omega})\rvert = \lvert A(\omega)\rvert$
4	Antisymmetric	Even	h (N-1-n) = - h(n)	$\left\lvert \sum_{n=1}^{\frac{N}{2}} 2\, h\left(\frac{N}{2}-n\right) \sin\left(\omega\left(n-\frac{1}{2}\right)\right) \right\rvert$
5	Symmetric	Odd	h(-n) = h(n)	$\left\lvert h(0) + \sum_{n=1}^{\frac{N-1}{2}} 2\, h(n) \cos\omega n \right\rvert$
6	Antisymmetric	Odd	h(-n) = -h(n)	$\left\lvert \sum_{n=1}^{\frac{N-1}{2}} 2\, h(n) \sin\omega n \right\rvert$

LINEAR PHASE FIR FILTERS

- **Magnitude and Phase function of FIR filter when impulse response is symmetric and N is odd:**

Magnitude Response

$$|H(e^{j\omega})| = \left| h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{N-1} 2h\left(\frac{N-1}{2} - n\right)\cos\omega n \right|$$

Phase Response

$$\angle H(e^{j\omega}) = \theta(\omega) = -\alpha\omega$$

- **Magnitude and Phase function of FIR filter when impulse response is symmetric and N is even:**

Magnitude Response

$$|H(e^{j\omega})| = \left| \sum_{n=1}^{N/2} 2h\left(\frac{N}{2} - n\right) \cos\left(\omega\left(n - \frac{1}{2}\right)\right) \right|$$

Phase Response

$$\angle H(e^{j\omega}) = \theta(\omega) = -\alpha\omega$$

- **Magnitude and Phase function of FIR filter when impulse response is antisymmetric and N is odd:**

Magnitude Response

$$|H(e^{j\omega})| = \left| \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - n\right) \sin\omega n \right|$$

Phase Response

$$\angle H(e^{j\omega}) = \theta(\omega) = \beta - \alpha\omega$$

- **Magnitude and Phase function of FIR filter when impulse response is antisymmetric and N is even:**

Magnitude Response

$$|H(e^{j\omega})| = \left| \sum_{n=1}^{\frac{N}{2}} 2h\left(\frac{N}{2} - n\right) \sin\left(\omega\left(n - \frac{1}{2}\right)\right) \right|$$

Phase Response

$$\angle H(e^{j\omega}) = \theta(\omega) = \beta - \alpha\omega$$

DESIGN TECHNIQUES FOR LINEAR PHASE FIR FILTERS

- **There are three well known method of design techniques for Linear phase FIR Filters**
- Fourier Series Method and Window method
- Frequency Sampling Method
- Optimal filter design methods

The following two concepts leads to the design of FIR Filters by Fourier Series Method

1. The frequency response of a digital filter is periodic with period equal to 2**π**
2. Any periodic function can be expressed as a linear combination of complex exponentials

DESIGN TECHNIQUES FOR LINEAR PHASE FIR FILTERS

- In frequency sampling method of filter design, begin with desired frequency response specification $H_d(e^{j\omega})$ and it is sampled at N –points to generate a sequence H(k)
- The N – Point inverse DFT of the sequence H(k) gives the impulse response of the filter h(n). The Fourier transform of h(n) gives the frequency response $H(e^{j\omega})$ and Z transform of h(n) gives the transfer function H(z) of the filter
- The FIR filter design by window and frequency sampling method does not have precise control over the critical frequencies such as ω_p (Pass band edge frequency) and ω_s (Stop band edge frequency). This drawback can be overcome by using Chebyshev approximation technique.

DESIGN TECHNIQUES FOR LINEAR PHASE FIR FILTERS

- In this method, the weighed approximation error between the desired frequency response and the actual frequency response is spread evenly across the pass band and evenly across the stop band of the filter. This results in the reduction of maximum error
- The resulting filter have ripples in both the pass band and stop band. This concept of design is called **Optimum Equiripple Design Criterion**
- **Gibbs Phenomenon (or) Gibbs Oscillation:** IN FIR filter design by Fourier series method, the infinite duration impulse response is truncated to finite duration impulse response. The abrupt truncation of impulse response introduces oscillations in the pass band and the stop band

SPECIFICATIONS FOR FIR FILTER DESIGN BY FOURIER SERIES METHOD

Low Pass

$$H_d(e^{j\omega}) = \begin{cases} 1 \; ; & \text{for} \quad -\omega_c \le \omega \le +\omega_c \\ 0 \; ; & \text{for} \quad -\pi \le \omega < -\omega_c \\ 0 \; ; & \text{for} \quad \omega_c < \omega \le \pi \end{cases}$$

High Pass

$$H_d(e^{j\omega}) = \begin{cases} 1 \; ; & \text{for} \quad -\pi \le \omega \le -\omega_c \\ 1 \; ; & \text{for} \quad \omega_c \le \omega \le \pi \\ 0 \; ; & \text{for} \quad -\omega_c < \omega < +\omega_c \end{cases}$$

SPECIFICATIONS FOR FIR FILTER DESIGN BY FOURIER SERIES METHOD

Band Pass

$$H_d(e^{j\omega}) = \begin{cases} 1 \ ; & \text{for} \quad -\omega_{c2} \le \omega \le -\omega_{c1} \\ 1 \ ; & \text{for} \quad \omega_{c1} \le \omega \le \omega_{c2} \\ 0 \ ; & \text{for} \quad -\pi \le \omega < -\omega_{c2} \\ 0 \ ; & \text{for} \quad -\omega_{c1} < \omega < +\omega_{c1} \\ 0 \ ; & \text{for} \quad \omega_{c2} < \omega \le \pi \end{cases}$$

Band Stop

$$H_d(e^{j\omega}) = \begin{cases} 1 \ ; & \text{for} \quad -\pi \le \omega \le -\omega_{c2} \\ 1 \ ; & \text{for} \quad -\omega_{c1} \le \omega \le +\omega_{c1} \\ 1 \ ; & \text{for} \quad \omega_{c2} \le \omega \le \pi \\ 0 \ ; & \text{for} \quad -\omega_{c2} < \omega < -\omega_{c1} \\ 0 \ ; & \text{for} \quad \omega_{c1} < \omega < \omega_{c2} \end{cases}$$

DESIRED IMPULSE RESPONSE FOR FIR FILTER DESIGN BY FOURIER SERIES METHOD

Low Pass

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega = \frac{1}{2\pi} \int_{-\omega_c}^{+\omega_c} e^{j\omega n}\, d\omega$$

$$\left[\because H_d(e^{j\omega}) = 0 \text{ in the range } -\pi \le \omega < -\omega_c \text{ and } +\omega_c < \omega \le \pi\right]$$

High Pass

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega = \frac{1}{2\pi} \int_{-\pi}^{-\omega_c} e^{j\omega n}\, d\omega + \frac{1}{2\pi} \int_{\omega_c}^{\pi} e^{j\omega n}\, d\omega$$

$$\left[\because H_d(e^{j\omega}) = 0 \text{ in the range } -\omega_c < \omega < +\omega_c\right]$$

DESIRED IMPULSE RESPONSE FOR FIR FILTER DESIGN BY FOURIER SERIES METHOD

Band Pass

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega = \frac{1}{2\pi}\int_{-\omega_{c2}}^{-\omega_{c1}} e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\omega_{c1}}^{\omega_{c2}} e^{j\omega n}\, d\omega$$

$$\left[\because H_d(e^{j\omega}) = 0 \text{ in the range } -\pi \le \omega < -\omega_{c2}\ ;\ -\omega_{c1} < \omega < +\omega_{c1} \text{ and } +\omega_{c2} < \omega \le \pi\right]$$

Band Stop

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega = \frac{1}{2\pi}\int_{-\pi}^{-\omega_{c2}} e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{-\omega_{c1}}^{\omega_{c1}} e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\omega_{c2}}^{+\pi} e^{j\omega n}\, d\omega$$

$$\left[\because H_d(e^{j\omega}) = 0 \text{ in the range } -\omega_{c2} < \omega < -\omega_{c1} \text{ and } +\omega_{c1} < \omega < +\omega_{c2}\right]$$

PROCEDURE FOR DIGITAL FIR FILTER BY FOURIER SERIES METHOD

1. The specifications of digital FIR filter are,

(i) The desired frequency response $H_d(e^{j\omega})$

(ii) The cutoff frequency ω_c for lowpass and high pass ω_{c1} and ω_{c2} for bandpass and bandstop filters.

(iii) The number of samples of impulse response N

2. Determine the desired impulse response $h_d(n)$ by taking inverse Fourier transform of the desired frequency response $H_d(e^{j\omega})$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

PROCEDURE FOR DIGITAL FIR FILTER BY FOURIER SERIES METHOD

3. Calculate N samples of $h_d(n)$ for n= -(N-1)/2 to (N-1)/2 and form the impulse response h(n) of FIR filter

Impulse Response

$$h(n) = h_d(n)\Big|_{n=-\frac{N-1}{2} \text{ to } +\frac{N-1}{2}}$$

- The impulse response is symmetric with n=0 and h(-n) =h(n). Calculate h(n) for n=0 to (N-1)/2

4. Take Z transform of the impulse response to get the noncausal transfer function of FIR Filter $H_N(z)$

$$H_N(z) = \mathcal{Z}\{h(n)\} = \sum_{n=-\frac{N-1}{2}}^{+\frac{N-1}{2}} h(n)\, z^{-n}$$

5. Convert the noncausal transfer function, $H_N(z)$ to causal transfer function, H(z) by multiplying $H_N(z)\, Z^{-(N-1)/2}$

$$H(z) = z^{-\frac{N-1}{2}} \sum_{n=-\frac{N-1}{2}}^{+\frac{N-1}{2}} h(n) z^{-n}$$

Transfer Function

$$H(z) = z^{-\frac{N-1}{2}} \left[h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{n} + z^{-n}\right] \right]$$

6. Draw a suitable structure for realization of FIR filter

ASSESSMENT

1. Define FIR Systems.
2. Mention the advantages and disadvantages of FIR Filters.
3. Based on frequency response the filters are classified into four basic types. They are ------------, --------------, ------------------- and ----------------
4. What are the steps involved in designing FIR Filter?
5. Summarize the frequency response characteristics of linear phase FIR filters.
6. In order to examine the linear and nonlinear phase characteristics, two delay functions are -------------- and ---------------
7. The Fourier transform of h(n) is ----------------------

THANK YOU