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Type - III $\quad f(z, p, q) = 0.$

1. Solve: $p(1+q) = qz$

Soln: $p(1+q) = qz \rightarrow (1)$

Let $u = x + ay$

then $p = \frac{dz}{\partial u}$ and $q = a\frac{dz}{du}$

$(1) \Rightarrow \frac{dz}{du}\left(1 + a\frac{dz}{du}\right) = a\frac{dz}{du} z$

$$1 + a\frac{dz}{du} = az$$

$$a\frac{dz}{du} = az - 1$$

$$\frac{dz}{du} = \frac{az-1}{a} \Rightarrow \frac{du}{dz} = \frac{a}{az-1}$$

$$du = \frac{a}{az-1} dz$$

Integrating, $\int du = \int \frac{a}{az-1}\, dz$

$$u = \log(az-1) + \log c$$

$$x+ay = \log(c(az-1))$$

2) Solve: $z^2 = 1+p^2+q^2$

Soln: $z^2 = 1+p^2+q^2 \rightarrow (1)$

Let $u = x+ay$ $\quad p = \frac{dz}{du}$, $q = a\frac{dz}{du}$

$$(1) \Rightarrow z^2 = 1+\left(\frac{dz}{du}\right)^2 + \left(a\frac{dz}{du}\right)^2$$

$$z^2 = 1+\left(\frac{dz}{du}\right)^2(1+a^2)$$

$$z^2-1 = \left(\frac{dz}{du}\right)^2(1+a^2$$

$$\left(\frac{dz}{du}\right)^2 = \frac{z^2-1}{1+a^2} \Rightarrow \frac{dz}{du} = \sqrt{\frac{z^2-1}{1+a^2}} = \frac{\sqrt{z^2-1}}{\sqrt{1+a^2}}$$

$$\frac{dz}{\sqrt{1-z^2}} = \frac{du}{\sqrt{1+a^2}}$$

Integrating on both sides

$$\cosh^{-1} z = \frac{1}{\sqrt{1+a^2}}u + c = \frac{1}{\sqrt{1+a^2}}(x+ay)+c$$

Type-IV $f_1(x,p) = f_2(y,q)$

for this type, there is no Singular Integral.

1- Solve $q^2 - p = y-x$

Given: $q^2-p = y-x$ ~~eeeke~~ (constant)

$q^2-y = p-x = k$

Now, $q^2 - y = k$ $\quad\mid\quad p-x = k$

$q^2 = k+y$ $\quad\mid\quad p = k+x$

$q = \sqrt{k+y}$