18 Steam Nozzles

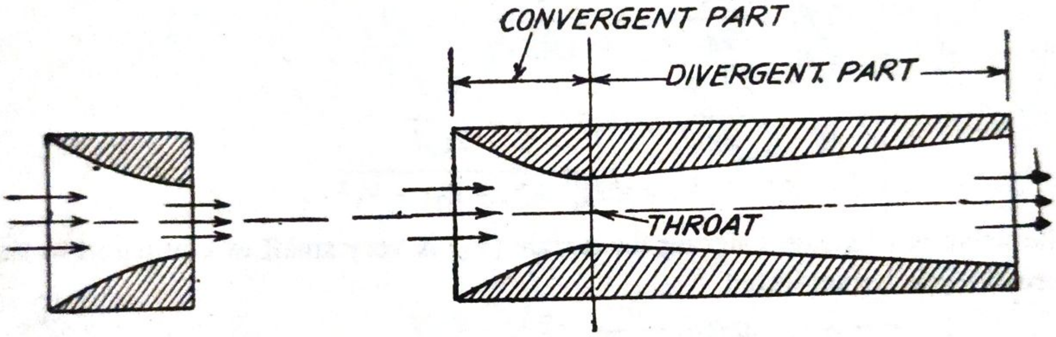
18.1. INTRODUCTION

The main purpose of steam nozzles is to produce a high velocity jet of steam which is used in steam turbines. Also, these are used in injectors which are used for pumping feed water into boilers. Nozzles are also used in injectors to maintain high vacuum in power plant condensers or steam jet refrigeration condensers.

The steam nozzle is a passage of varying cross-section by means of which a part of the enthalpy of steam is converted into kinetic energy as the steam expands from a higher pressure to a lower pressure. The amount of energy so converted depends upon the pressure ratio and the type of expansion. Isentropic expansion provides the maximum conversion. Generally nozzles are so shaped that isentropic expansion is obtained.

18.2. TYPES OF NOZZLES

There are mainly two types of nozzles as shown in Fig. 18.1.



Convergent Nozzle

Convergent divergent Nozzle

Fig. 18.1.

Convergent Nozzle

The cross-sectional area of the nozzle is dictated by the expansion process and the condition of the steam at inlet and at exit.

In convergent nozzles, the cross-sectional area diminishes from the inlet section to the outlet section. This type of nozzle is useful up to a pressure ratio of 0.58 when using saturated steam.

This is known as critical pressure ratio and is different for different fluids.

Convergent-Divergent Nozzle

The convergent-divergent nozzle is also shown in Fig. 18.1. When the pressure ratio has a value less than the critical value, a divergent part is necessary in addition to the convergent portion to obtain further pressure drop and acceleration. The divergent section has to be long as the divergent angle is limited to about 7° in order to prevent separation at the wall.

The least cross-section of convergent-divergent nozzle is known as the throat of the nozzle, and it corresponds to the limiting exit section of a convergent nozzle. Beyond this section, the nozzle area increases till the exit section.

The design of correct areas at the throat and exit of the nozzle is very important as the flow through the nozzle will depend upon these sizes for a given pressure drop. Nozzles are always designed to discharge maximum mass for a given set of conditions.

18.3. FLOW OF STEAM THROUGH NOZZLE

Consider a nozzle as shown in Fig. 18.2. Applying the energy equation to the sections 1 and 2 of the convergent nozzle and considering a flow rate of one kg per second,

$$h_1 + \frac{V_1^2}{2g_cJ} W \pm Q = h_2 + \frac{V_2^2}{2g_cJ} \qquad ...(a)$$

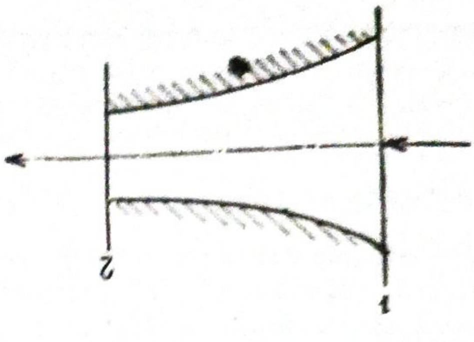


Fig. 18.2.

where h and V denote enthalpy and velocity of steam and W and Q are work and heat transfers. g_c has a numerical value of 1 in SI units.

Since, the expansion through a nozzle is considered as isentropic, and, as there is no external work done during the flow of steam, both the heat transfer and work transfer have zero values

$$Q = 0 \quad ...(b)\ ; \qquad W = 0 \quad ...(c)$$

Using these conditions, equation (a) simplifies to

$$h_1 + \frac{V_1^2}{2g_cJ} = h_2 + \frac{V_2^2}{2g_cJ}$$

$$\therefore \qquad \frac{V_2^2}{2g_cJ} = (h_1 - h_2) + \frac{V_1^2}{2g_cJ}$$

i.e.

$$V_2 = \sqrt{2g_c\ J(h_1 - h_2) + V_1^2} \qquad ...(18.1)$$

Usually, the velocity of steam entering the nozzle (V_1) is very small as compared to the velocity at exit, and therefore, V_1 can be neglected.

Otherwise the quantity $\left(h_1 + \frac{V_1^2}{2g_cJ}\right) = h_0$

may be used where h_0 is known as the stagnation enthalpy at inlet.

$$\therefore \qquad V_2 = \sqrt{2g_cJ(h_1 - h_2)} = \sqrt{2g_cJ\ (\Delta\ h)_{ise}}$$

$$= \sqrt{2 \times 1(\Delta h)_{isc} \times 10^3 \times 1} \text{ where } (\Delta h)_{sic} \text{ is in kJ/kg} \qquad ...(18.2)$$

$$= 44.72\ \sqrt{\Delta h_{ise}} \text{ As } g_c = 1 \text{ and } J = 1 \qquad \Delta h \text{ in kJ}$$

where Δh_{ise} is the isentropic enthalpy drop per kg of steam when the pressure drop is from p_1 to p_2.

This is the general energy equation irrespective of the shape of the nozzle.

The isentropic flow of steam through the nozzle may be approximately represented by an equation of the form

$$pv^n = \text{constant}$$

where $n = 1.135$ for saturated steam and $= 1.3$ for superheated steam.

These values are approximate and actually the value of n varies along the nozzle.

It can be said that the steam performs work upon itself by accelerating itself to a high velocity, instead of doing work on the piston of an engine. It is a reversible expansion.

As the steam pressure drops while passing through the nozzle, its enthalpy is reduced. This reduction of enthalpy of the steam must be equal to the increase in kinetic energy as given by equation 18.1. Hence the work done by the steam upon itself is equal to the enthalpy drop.

If the law of expansion is assumed to be pv^n = constant, the work done during the expansion will be equal to the work done during the flow process as mentioned earlier.

i.e.
$$\frac{V_2^2}{2g_c} - \frac{V_1^2}{2g_c} = (p_1v_1 - p_2v_2)\frac{n}{n-1}$$

where

v_1 = (specific volume of steam at entry)

v_2 = specific volume of steam at exit)

If $V_1 << V_2$, then

$$\frac{V_2^2}{2g_c} = \frac{n}{n-1}\left(p_1v_1 - p_2v_2\right) = \frac{n}{n-1}p_1v_1\left(1 - \frac{p_2v_2}{p_1v_1}\right)$$

But
$$p_1v_1^n = p_2v_2^n$$

$\therefore$
$$\frac{v_2}{v_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}$$

Substituting this in the above equation, we get

$$\frac{V_2^2}{2g_c} = \frac{n}{n-1}p_1v_1\left[1 - \frac{p_2}{p_1}\cdot\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}\right]$$

$$= \frac{n}{n-1}p_1v_1\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]$$

$\therefore$
$$V_2 = \sqrt{2g_c\frac{n}{n-1}p_1v_1\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]} \quad ...(18.3)$$

18.4. CONDITION FOR MAXIMUM DISCHARGE

Nozzles are always designed for maximum discharge.

The flow of steam through the nozzle is given by

$$m_s = \frac{A_2V_2}{v_2} \quad ...(18.4)$$

where

m_s = Mass of steam passing through the nozzle in kg per second.

A_2 = Area at exit of nozzle in m^2.

V_2 = Velocity at exit in m/sec.

v_2 = Volume of one kg of steam at exit of nozzle in m^3.

Substituting the value of V_2 from equation 18.3 into equation 18.4.

$$m_s = \frac{A_2}{v_2}\sqrt{2g_c\frac{n}{n-1}p_1v_1\left[1 - \left(\frac{p_2}{p_1}\right)\right]^{\frac{n-1}{n}}}$$

But
$$p_1v_1^n = p_2v_2^n$$

$\therefore$
$$v_2 = v_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = v_1\left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}}$$

Substituting this value of v_2 in the above equation,

$$m_s = \frac{A_2}{v_1}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} \sqrt{2g_c \frac{n}{n-1} p_1 v_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]}$$

$$= \frac{A_2}{v_1}\sqrt{g_c \cdot \frac{n}{n-1} p_1 v_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right]} \quad \text{...(18.5)}$$

The equation 18.5 gives the discharge of steam through the converging nozzle. It is obvious from this equation that the discharge through given nozzle is mainly a function of $\frac{p_2}{p_1}$, as expansion index n is fixed according to the quality of steam supplied to the nozzle.

Therefore the condition for maximum discharge of steam through the nozzle is obtained as outlined below.

$$\frac{dm_s}{d\left(\frac{p_2}{p_1}\right)} = 0$$

or

$$\frac{d}{d\left(\frac{p_2}{p_1}\right)}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right] = 0$$

i.e.,

$$\frac{2}{n}\left(\frac{p_2}{p_1}\right)^{\left(\frac{2}{n}-1\right)} - \frac{n+1}{n}\left(\frac{p_2}{p_1}\right)^{\left(\frac{n+1}{n}-1\right)} = 0$$

or

$$\frac{2}{n}\left(\frac{p_2}{p_1}\right)^{\frac{2-n}{n}} = \frac{n+1}{n}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

or

$$\frac{2}{n}\cdot\frac{n}{n+1} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}\left(\frac{p_2}{p_1}\right)^{-\frac{(2-n)}{n}}$$

or

$$\frac{2}{n+1} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}-\frac{(2-n)}{n}} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$\therefore$

$$\left(\frac{p_2}{p_1}\right) = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}} \quad \text{...(18.6)}$$

This pressure ratio which gives maximum discharge through the nozzle is known as the critical pressure ratio.

Now, $n = 1.135$ for saturated steam

and $n = 1.3$ for superheated steam

Substituting these values in the above equation

We get $p_2 = 0.58\ p_1$ for saturated steam

and $p_2 = 0.5457\ p_1$ for superheated steam.

Substituting the value of $\left(\frac{p_2}{p_1}\right)$ from equation (18.6) into equation 18.5, we get

The maximum discharge m_{max}

$$= \frac{A_1}{v_1}\sqrt{2g_c\,\frac{n}{n-1}\,p_1v_1\left[\left\{\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}\right\}^{\frac{2}{n}}-\left\{\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}\right\}^{\frac{n+1}{n}}\right]}$$

$$= \frac{A_2}{v_2}\sqrt{2g_c\,\frac{n}{n-1}\,p_1v_1\left[\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}-\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\right]}$$

$$= A_2\sqrt{2g_c\,\frac{n}{n-1}\,\frac{p_1}{v_1}\left[\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}-\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\right]}$$

$$= A_2\sqrt{2g_c\,\frac{n}{n-1}\cdot\frac{p_1}{v_1}\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\left[\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}-\frac{n+1}{n-1}}-1\right]}$$

$$= A_2\sqrt{2g_c\,\frac{n}{n-1}\left(\frac{p_1}{v_1}\right)\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\left[\left(\frac{2}{n+1}\right)^{\frac{1-n}{n-1}}-1\right]}$$

$$= A_2\sqrt{2g_c\,\frac{n}{n-1}\left(\frac{p_1}{v_1}\right)\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\left[\left(\frac{2}{n+1}\right)^{-1}-1\right]}$$

$$= A_2\sqrt{2g_c\,\frac{n}{n-1}\left(\frac{p_1}{v_1}\right)\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\left(\frac{n-1}{2}\right)}$$

$$\therefore \quad (m_s)_{max} = A_2\sqrt{g_c\,n\left(\frac{p_1}{v_1}\right)\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}} \quad ...(18.7)$$

It is obvious from the above equation that the maximum mass flow depends only on the initial condition of the steam (p_1, v_1) and the throat area and it is independent of the final pressure of steam. *i.e.*, at the exit of the nozzle. The addition of the divergent part of the nozzle after the throat does not affect the discharge of steam passing through the nozzle but it only accelerates the steam leaving the nozzle.

The velocity of steam at the throat of the nozzle when the discharge is maximum is obtained by substituting the value of $\frac{p_2}{p_1}$ from equation 18.6 into equation 18.3.

$$(V_2)_{max} = \sqrt{2g_c \frac{n}{n-1} p_1 v_1 \left[1 - \left\{\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}\right\}^{\frac{n-1}{n}}\right]}$$

$$= \sqrt{2g_c \frac{n}{n-1} p_1 v_1 \left[1 - \frac{2}{n+1}\right]} = \sqrt{2g_c \frac{n}{n-1} p_1 v_1 \left(\frac{n-1}{n+1}\right)}$$

i.e.

$$(V_2)_{max} = \sqrt{2g_c \frac{n}{n+1} p_1 v_1} \qquad \text{...(18.}$$

This is also found to be dependent on the initial conditions of the steam only.

The variations of mass flow, steam velocity and specific volume of steam with respect to the bac pressure for a convergent-divergent nozzle is shown in Fig. 18.3.

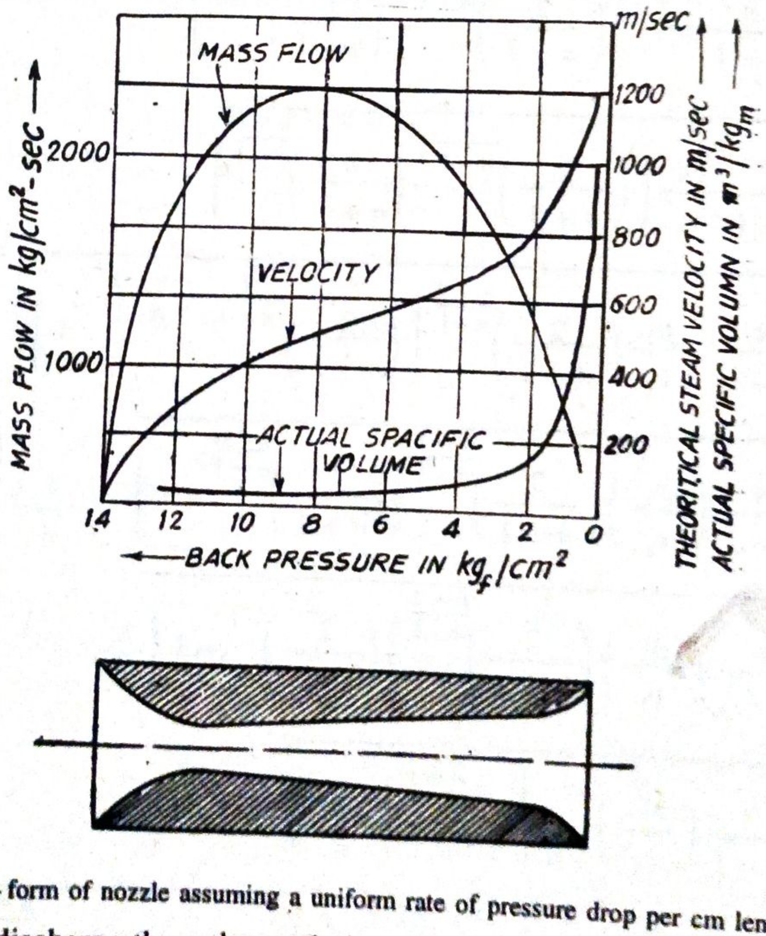


Fig. 18.3. Theoretical form of nozzle assuming a uniform rate of pressure drop per cm length of nozzle.

It is obvious that the discharge through nozzle increases as the pressure at the throat of the nozzle (p_2) decreases, when the supply pressure p_1 is constant. But once the nozzle exit pressure p_2 reaches the critical value given by the equation (18.6), the discharge reaches a maximum and after that the throat pressure and mass flow remain constant irrespective of the pressure at the exit.