... irrespective of the pressure at the exit. ... that the throat pressure

18.5. EXPANSION OF STEAM CONSIDERING FRICTION

The exit velocity of steam for a given pressure drop is reduced due to the following reasons when the steam flows through the nozzle.

1. Due to the friction between the nozzle surface and steam.
2. Due to the internal fluid friction in the steam.
3. Due to shock losses.

Most of these frictional losses occur beyond the throat in the divergent section nozzles as the length as well as the velocity of steam in the divergent portion is much higher.

The effects of these frictional losses are listed below :

1. The expansion is no more isentropic and the enthalpy drop is reduced resulting in lower exit velocity.

2. The final dryness fraction of the steam is increased as part of the kinetic energy gets converted into heat due to friction and is absorbed by the steam, with increase in enthalpy.

3. The specific volume of steam is increased as the steam becomes more dry due to this frictional reheating.

The effect of friction on steam flow through a nozzle is represented on Mollier diagram as shown in Fig. 18.4.

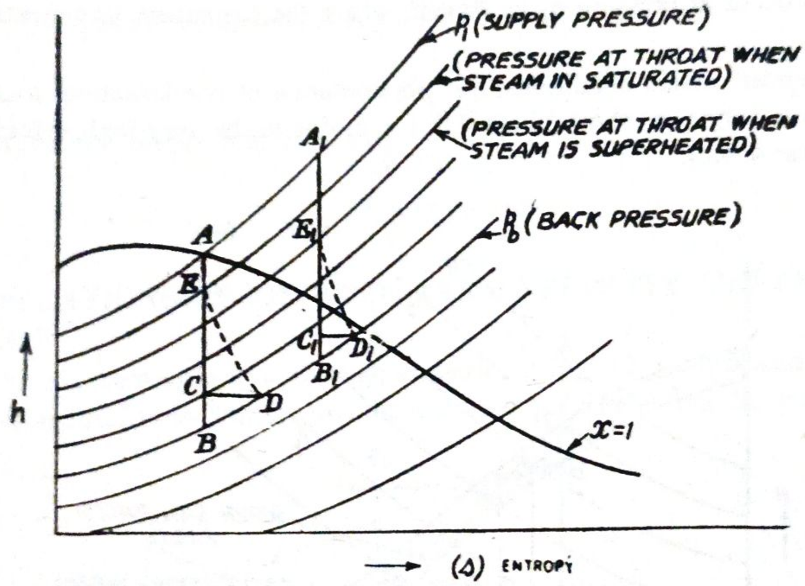

Fig. 18.4. Effect of friction on the steam quality in flow through a nozzle.

The point A represents the initial condition of steam and the point E is the steam condition at the throat of a convergent-divergent nozzle. If the expansion through the nozzle is isentropic, then it is represented by the vertical line AB as shown. Due to friction, the process is now represented by AC and the enthalpy drop is reduced to $(h_A - h_C)$ instead of $(h_A - h_B)$, which is theoretically possible. Then $(h_A - h_C)$ is the useful heat drop causing the increase in velocity of steam from V_1 to V_2. If the actual enthalpy drop as percentage of theoretical enthalpy drop is known, the point C can be located. But the expansion must end at same pressure as at B. The line drawn through C parallel to X-axis cuts the pressure line P_b at point D. Therefore, point D represents the final condition of the steam. It is obvious from the diagram that the dryness fraction of steam at point D is greater than at point B and therefore the effect of friction is to dry out the steam leaving the nozzle and to increase the specific volume of the steam.

Actually, most of the friction occurs between the throat and exit (divergent part of the nozzle) and therefore the actual expansion is represented by the line AED. AE represents the expansion through the convergent part of the nozzle (the effect of friction in this part of nozzle is negligible) and ED represents the actual expansion through the divergent part of the nozzle.

The effect of friction is taken into account by introducing a factor called "nozzle efficiency" which is defined as follows :

$$\eta_n(\text{nozzle efficiency}) = \frac{\text{Actual enthalpy drop}}{\text{Isentropic enthalpy drop}} = \frac{h_A - h_D}{h_A - h_B} \qquad ...(18.9)$$

The exit velocity considering friction is given by

$$V_2 = 44.72\sqrt{\eta_n(\Delta h_{ise})} \text{ where, } \Delta h \text{ in kJ} \quad ...(18.10)$$

It can also be given by an equation

$$V_2 = 44.72\, K\sqrt{\Delta\, h_{ise}} \text{ where } \Delta h \text{ in kJ}$$

and K is the coefficient of the nozzle, where

$$K = \sqrt{\eta_n}$$

18.6. SUPERSATURATED OR METASTABLE FLOW THROUGH NOZZLE

The ideal expansion of superheated steam from pressure p_1 (supply pressure) to p_b (back pressure) can be represented by a line AE on Mollier diagram as shown in Fig. 18.5. During the expansion, the change of phase must start to occur at pressure p_2 as shown, where the expansion line meets the saturation line (that is at point B).

But in nozzles, under certain conditions, this phenomenon of condensation does not occur at point B as the time available is very very short (about 0.001 sec) due to the very high velocity of steam passing through the nozzle (near sonic).

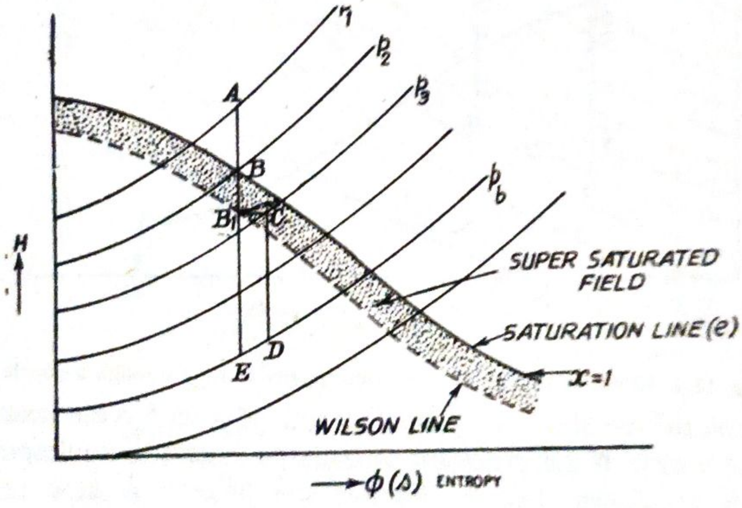

Fig. 18.5. Supersaturated flow of steam.

The equilibrium between the liquid and vapour phase is therefore delayed and the vapour continues to expand in dry state even beyond point B. This is represented by BB_1 in Fig. 18.5. The pressure at the point B_1 can be found by extending the superheated constant pressure line (p_3) upto B_1 as shown in diagram by dotted line. The steam during the expansion BB_1 remains dry and condensation is suppressed.

The vapour between the pressure p_2 and p_3 is said to be supersaturated or supercooled and this type of flow in nozzles is known as supersaturated or metastable flow of steam.

A limit to the supersaturated state was observed by Wilson and a line drawn on the chart through the observed points is known as Wilson line. This line becomes the saturation line for all practical purposes.

The flow is also called supercooled flow because at any pressure between p_2 and p_3 the temperature of the vapour is always lower than the saturation temperature corresponding to that pressure. The difference in this temperature is known as the *degree of under-cooling.*

When the expansion reaches the Wilson line (as shown by point B_1), the condensation occurs at constant enthalpy, the pressure remaining constant. This is shown by B_1C. Further isentropic expansion to the exit pressure represented by CD.

The ratio of saturation pressures corresponding to the temperature at B and B_1 is known as the degree of supersaturation. During the process B_1C, the partial condensation of steam releases sufficient heat to raise the temperature of the steam back to the saturation temperature.

The problems on supersaturated flow cannot be solved by using Mollier Chart unless Wilson line is drawn on it.

The velocity of steam at the end of expansion is found by using

$$V_2 = \sqrt{2 \cdot \frac{n}{n-1} \cdot p_1 v_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right]}$$

$$\text{Specific volume } v_2 = v_1 \left[\frac{P_1}{P_2}\right]^{\frac{1}{n}}$$

$$\text{Apparent temperature } T_2 = T_1 \left[\frac{P_2}{P_1}\right]^{\frac{n-1}{n}}$$

$$\text{and} \quad A_2 = \frac{m \cdot v_2}{V_2}.$$

18.7. GENERAL RELATIONSHIP BETWEEN AREA, VELOCITY AND PRESSURE IN NOZZLE FLOW

Consider steady and isentropic flow through a nozzle. In Fig. 18.6, we consider two transverse plane sections, a distance δx apart. Let us assume that the nozzle runs full and that the velocity is uniform across any section.

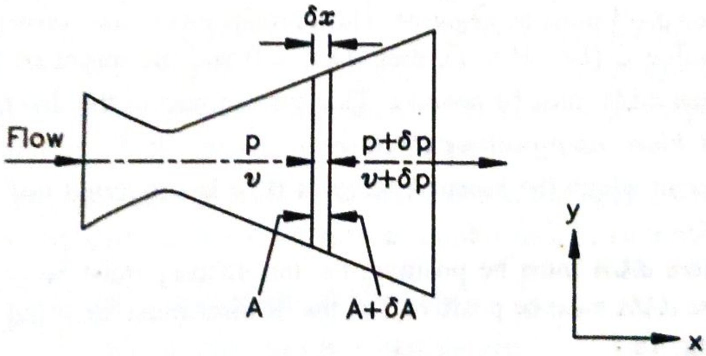

Fig. 18.6.

Then, by continuity equation, $$m = \frac{A \cdot V}{v} = \frac{(A + \delta A)(V + \delta V)}{v + \delta v}$$

from which, we get $$\frac{\delta A}{A} + \frac{\delta V}{V} - \frac{\delta v}{v} = 0$$

and in limits $$\frac{dA}{A} + \frac{dV}{V} - \frac{dv}{v} = 0 \qquad \text{...(18.11)}$$

Since the flow is isentropic $\quad pv^\gamma = \text{constant}$

$\therefore \quad \log_e p + \gamma \log_e v = \log_e C$

Differentiating this and dividing by pv

$$\frac{dp}{p} + \gamma \cdot \frac{dv}{v} = 0$$

$$\therefore \quad \frac{dv}{v} = -\frac{1}{\gamma} \cdot \frac{dp}{p}$$

Also, for isentropic flow, we have from the momentum equation :

$$\frac{VdV}{g_c} = -v\ dp$$

or

$$\frac{dV}{V} = -\frac{g_c v \cdot dp}{V^2}$$

By substituting these values in equation (18.11), we get

$$\frac{dA}{A} = \frac{1}{n} \cdot \frac{dp}{p}\left(\frac{g_c \gamma\ pv}{V^2} - 1\right)$$

and writing '*C*' for the sonic velocity at pressure *p* and specific volume *v*, we get (as sonic velocity is given by $C^2 = g_c \gamma RT$

$= g_c \gamma pv$ in this case

$$\frac{dA}{A} = \frac{1}{n} \cdot \frac{dp}{p}\left(\frac{C^2}{V^2} - 1\right) \qquad ...(18.12)$$

The ratio of the velocity '*V*' to thé local sonic velocity '*C*' is known as the Mach number and is denoted by the letter '*M*'.

$$\therefore \quad \frac{dA}{A} = \frac{1}{n} \cdot \frac{dp}{p}\left(\frac{1 - M^2}{M^2}\right) \qquad ...(18.13)$$

Equations (18.12) and (18.13) give a useful insight into the changes of nozzle area under certain conditions. These may be explained briefly as follows.

Case I. *Accelerated flow, dp/p negative (nozzle), pressure decreases along flow direction.*

(*a*) $V < C, M < 1$. Then *dA/A* must be negative. This corresponds to the convergent part of the nozzle. As soon as *V* reaches the value *C* (*i.e.*, $M = 1$), then $dA/A \approx 0$ and the throat of the nozzle is reached.

(*b*) $V > C, M > 1$. Then *dA/A* must be positive. This corresponds to the divergent part of the nozzle.

Case II. *Decelerated Flow, dp/p positive (Diffusers).*

This applies to diffuser in which the kinetic energy of flow is converted into pressure energy, (little application in steam turbine).

(*a*) $V > C, M > 1$. Here *dA/A* must be positive, *i.e.* the diffuser must be of divergent type.

(*b*) $V > C, M > 1$. Here *dA/A* must be positive, *i.e.*, the diffuser must be of the convergent type. These forms are summarized in Fig. 18.7.

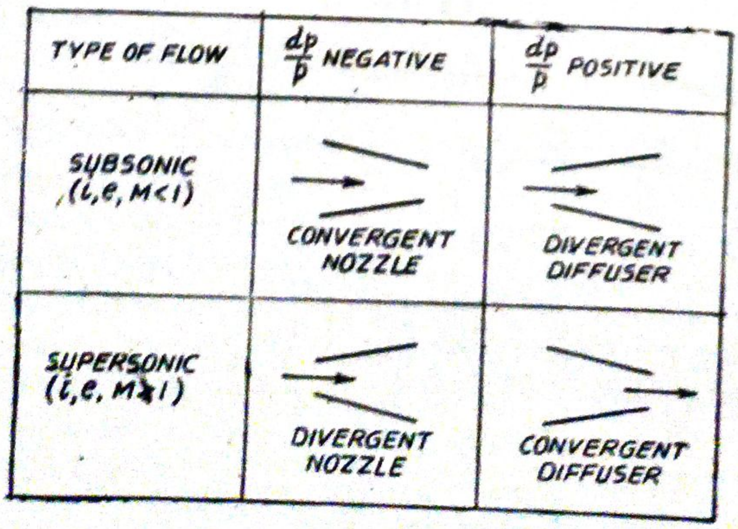

Fig. 18.7.