Central and up-wind type discretization

Central and upwind discretization are two common approaches used to approximate the spatial derivatives of the dependent variables in the finite volume method. They differ in how they handle the flux terms across cell faces. Here's an explanation of each approach:

Central Discretization:

In central differencing, the flux across a cell face is approximated using a symmetric difference scheme, where the flux at the face is computed as the average of the values on both sides of the face. Mathematically, for a scalar quantity ϕ, the central differencing scheme for the flux F across a cell face is given by:

$F=\Delta x\phi E-\phi W$

where ϕE and ϕW are the values of ϕ at the east and west faces of the cell, respectively, and Δx is the distance between the cell faces.

Advantages:

- Symmetric and straightforward.
- Can provide accurate results for problems with smooth solutions.

Limitations:

- Prone to numerical diffusion, especially in regions with steep gradients.
- May lead to oscillations or instability in convection-dominated problems.

Upwind Discretization:

In the upwind differencing scheme, the flux across a cell face is approximated using information from the upstream side of the flow. This means that the flux is calculated using the value of the dependent variable on the side from which the flow is coming. Mathematically, for a scalar quantity ϕ, the upwind differencing scheme for the flux F across a cell face is given by:

$F=\{\Delta x\phi P-\phi W\Delta x\phi E-\phi P$

if flow is from west to eastif flow is from east to west

where ϕP is the value of ϕ at the cell centroid (P), ϕE is the value at the east face, and ϕW is the value at the west face.

Advantages:

- Naturally captures the directionality of the flow.
- Minimizes numerical diffusion in convection-dominated problems.

Limitations:

- May introduce numerical oscillations or instability, especially in regions with discontinuities or sharp gradients.
- More complex to implement, especially for unsteady problems with changing flow directions.

Implementation:

- Both central and upwind discretization schemes can be implemented within the finite volume framework by appropriately calculating the flux terms across cell faces.

- The choice between central and upwind schemes depends on the characteristics of the problem, such as the flow regime (convection-dominated or diffusion-dominated) and the presence of discontinuities or steep gradients.

In summary, central and upwind discretization schemes are two common approaches used in finite volume methods to approximate spatial derivatives. Each scheme has its advantages and limitations, and the choice between them depends on the specific requirements and characteristics of the problem being solved.