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(An Autonomous Institution)

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Lax – Wendroff time stepping

The Lax-Wendroff method is a numerical scheme used for time integration in finite volume methods, particularly for hyperbolic partial differential equations (PDEs). Here's an explanation of how Lax-Wendroff time stepping works within the finite volume approach:

Overview:

1. **Finite Volume Discretization**:
   - The computational domain is discretized into control volumes, with cell-centered values of variables defined at the centroids of each cell. The conservation laws are then integrated over these control volumes to obtain discrete equations.
2. **Time Discretization**:
   - After spatial discretization, the next step is to discretize the equations in time. The Lax-Wendroff method is a second-order accurate time-stepping scheme that achieves this.
3. **Lax-Wendroff Time Stepping**:
   - The Lax-Wendroff method involves two steps:
     - **Predictor Step**: A predictor step calculates intermediate values of the variables at half-time levels using a Taylor series expansion.
     - **Corrector Step**: A corrector step updates the solution using the values obtained from the predictor step and a correction term to improve accuracy.
4. **Predictor Step**:
   - In the predictor step, the solution at the next time level $t+\Delta t\,t+\Delta t$ is estimated based on the current solution $t\,t$ and the time derivative terms.
   - The predictor step is typically implemented using a Taylor series expansion to approximate the solution at $t+\Delta t\,t+\Delta t$ based on the solution at $t\,t$.
5. **Corrector Step**:
   - The corrector step refines the prediction obtained in the predictor step by incorporating higher-order terms.
   - It corrects the solution by adding a term proportional to the second derivative with respect to time.
6. **Stability and CFL Condition**:
   - The stability of the Lax-Wendroff method is governed by the Courant-Friedrichs-Lewy (CFL) condition, which imposes a constraint on the time step size based on the spatial discretization and the speed of propagation of information in the system.

Advantages:

- **Second-Order Accuracy**: Provides higher accuracy compared to first-order methods like the forward Euler method.

- **Reduced Numerical Dispersion**: Helps in reducing numerical dispersion, particularly for wave-like phenomena.
- **Stability**: Can be conditionally stable for hyperbolic problems when the CFL condition is satisfied.

Limitations:

- **Diffusion**: Like any explicit method, the Lax-Wendroff scheme can introduce numerical diffusion, especially for highly oscillatory solutions.
- **CFL Condition**: The CFL condition can impose stringent restrictions on the time step size, particularly for problems with high wave speeds.

Implementation in Finite Volume Approach:

- In the context of finite volume methods, the Lax-Wendroff time stepping is applied to update the solution values within each control volume at each time step.
- The spatially discretized equations are advanced in time using the Lax-Wendroff method, incorporating flux terms and source terms if present.

Overall, the Lax-Wendroff time stepping scheme offers a good balance between accuracy and stability, making it a popular choice for time integration in finite volume simulations of hyperbolic PDEs. However, careful consideration of stability and numerical diffusion is necessary for its effective implementation.