Aileron effectiveness and reversal (2D-Case)

The case of a wing aileron combination in a two dimensional flow. An aileron deflection (ξ) produces changes ΔL and Δm_0 in the wing lift (L) and wing pitching moment M_0. These in turn cause an elastic twist θ of the wing. Thus,

$$\Delta L = \left(\frac{\partial C_L}{\partial \alpha} \theta + \frac{\partial C_L}{\partial \xi} \xi \right) \frac{1}{2} \rho V^2 S$$

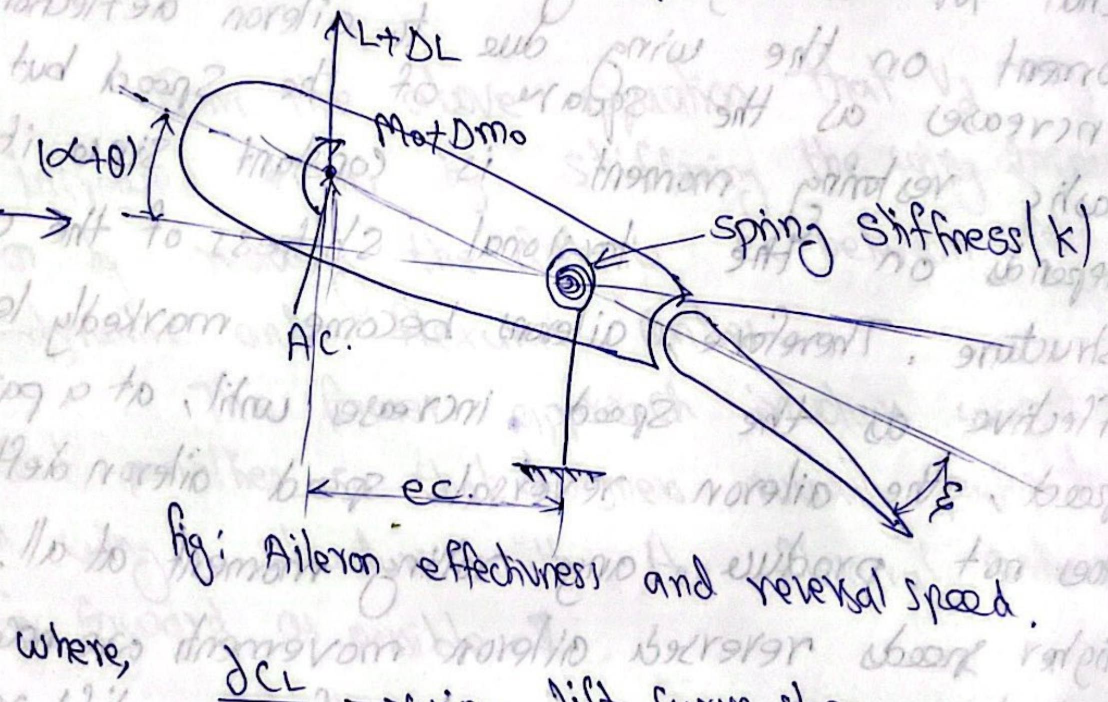

fig: Aileron effectiveness and reversal speed.

where, $\frac{\partial C_L}{\partial \alpha}$ → wing lift curve slope.

$\frac{\partial C_L}{\partial \xi}$ → rate of change of lift coefficient with aileron angle.

$$\Delta M_0 = \frac{\partial C_{M_0}}{\partial \xi} \xi \frac{1}{2} \rho V^2 S c$$

$\frac{\partial C_{M_0}}{\partial \xi}$ → rate of change of wing pitching moment coefficient with aileron deflection.

The moment produced by these increments in lift and pitching moment is equilibrated by an increment of Torque ΔT about the flexural axis, Hence

$$= \Delta L \times ec + \Delta M_0$$

$$\Delta T = K\theta = \frac{1}{2}\rho V^2 S\left[\frac{\partial C_L}{\partial \alpha}\theta + \frac{\partial C_L}{\partial \xi}\xi\right] \times ec + \frac{\partial C_{m_0}}{\partial \xi}\xi\,\frac{1}{2}\rho V^2 Sc.$$

$$K\theta = \frac{1}{2}\rho V^2 Sc\left[\left(\frac{\partial C_L}{\partial \alpha}\theta + \frac{\partial C_L}{\partial \xi}\xi\right)e + \frac{\partial C_{m,0}}{\partial \xi}\xi\right]$$

Isolating θ

$$\theta = \frac{\frac{1}{2}\rho V^2 Sc\left[(\partial C_L/\partial \xi)e + (\partial C_{m,0}/\partial \xi)\right]\xi}{K - \frac{1}{2}\rho V^2 Sce\left(\frac{\partial C_L}{\partial \alpha}\right)}$$

Sub θ in ΔL equation.

$$\Delta L = \frac{1}{2}\rho V^2 S\left[\frac{\frac{1}{2}\rho V^2 Sc\left[(\partial C_L/\partial \xi)e + (\partial C_{m,0}/\partial \xi)\right]}{K - \frac{1}{2}\rho V^2 Sce(\partial C_L/\partial \alpha)} \times \frac{\partial C_L}{\partial \alpha} + \frac{\partial C_L}{\partial \xi}\right]\xi.$$

which simplifies to.

$$\Delta L = \frac{1}{2}\rho V^2 S\left[\frac{\frac{1}{2}\rho V^2 Sc(\partial C_{m,0}/\partial \xi)(\partial C_L/\partial \alpha) + K(\partial C_L/\partial \xi)}{K - \frac{1}{2}\rho V^2 Sce(\partial C_L/\partial \alpha)}\right]\xi.$$

The increment of wing lift is therefore a linear function of aileron deflection and becomes zero, i.e. aileron reversal occurs, when—

$$\frac{1}{2}\rho V^2 S c \frac{\partial C_{m.o}}{\partial \xi}\frac{\partial C_L}{\partial \alpha} + K\frac{\partial C_L}{\partial \xi} = 0.$$

Hence aileron reversal speed V_r,

$$V_r = \sqrt{\frac{-K\left(\partial C_L/\partial \xi\right)}{\frac{1}{2}\rho S c\left(\frac{\partial C_{m.o}}{\partial \xi}\right)\left(\frac{\partial C_L}{\partial \alpha}\right)}}$$

We may define aileron effectiveness at speeds below the reversal speed in terms of the lift ΔL_R produced by an aileron deflection on a rigid wing.

$$\text{Aileron effectiveness} = \frac{\Delta L}{\Delta L_R}.$$

where,

$$\Delta L_R = \frac{\partial C_L}{\partial \xi}\xi \frac{1}{2}\rho V^2 S$$

$$\text{Aileron effectiveness} = \frac{\frac{1}{2}\rho V^2 S c\left(\frac{\partial C_{m.o}}{\partial \xi}\right)\left(\frac{\partial C_L}{\partial \alpha}\right) + K\left(\frac{\partial C_L}{\partial \xi}\right)}{\left[K - \frac{1}{2}\rho V^2 S c e\left(\frac{\partial C_L}{\partial \alpha}\right)\right]\frac{\partial C_L}{\partial \xi}}.$$

This eqn expressed in terms of the wing divergence speed V_d and aileron reversal speed V_r

$$\text{Aileron effectiveness} = \frac{1 - V^2/V_r^2}{1 - V^2/V_d^2}.$$

when $V_d = V$, which occurs when $\frac{\partial C_L}{\partial \xi} = \frac{\left(\frac{\partial C_{m,0}}{\partial \xi}\right)}{e}$

then the aileron is completely effective at all speeds. Such a situation arises.

Introduction to flutter

We have previously defined flutter as the dynamic instability of an elastic body in an airstream. It is found most frequently in aircraft structures subjected to large aerodynamic loads such as wings, tail units and control surfaces.

Flutter occurs at a critical or flutter speed V_f which in turn is defined as the lowest airspeed at which a given structure will oscillate with sustained simple harmonic motion. Flight at speeds below and above the flutter speed represents conditions of stable and unstable structural oscillation, respectively.

Generally, an elastic system having just one degree of freedom cannot be unstable unless some mechanical characteristic exists such as a negative spring force or a negative damping force. However, it is possible for systems with two or more degrees of freedom to be unstable without possessing unusual characteristi