Wing Torsional divergence (Two Dimensional Case)

The most common divergence problem is the Torsional divergence of a wing. It is useful initially to consider the case of a wing of area S without ailerons and in a two-dimensional flow, as shown in fig.

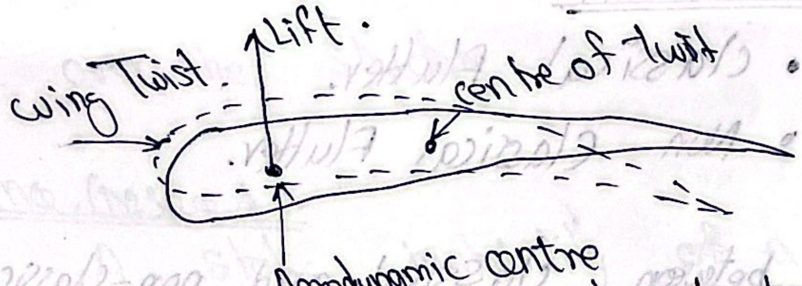

Fig: increase of wing incidence due to wing twist

The Torsional stiffness of the wing, which we shall represent by a spring of stiffness K. Resists the moment of the lift vector, L and wing pitching moment. M_0 acting at the aerodynamic centre of the wing section. For moment equilibrium of the wing section about the aerodynamic centre we have.

$$M_0 + Lec = K\theta$$

where, ec is the distance of the aerodynamic centre forward of the flexural centre expressed in terms of the wing chord c and θ is the elastic twist of the wing. From the aerodynamic theory.

$$M_0 = \frac{1}{2} \rho V^2 S c \, C_{M_0}$$

$$L = \frac{1}{2} \rho V^2 S C_L.$$

$$\therefore \frac{1}{2}\rho V^2 S\left[cC_{m_0} + ecC_L\right] = K\theta$$

(or)

$$C_L = C_{L_0} + \frac{\partial C_L}{\partial \alpha}(\alpha+\theta)$$

in which α is the initial wing incidence (or) in other words the incidence corresponding to given flight condition assuming that the wing is rigid and C_{L_0} is the wing lift coefficient at zero incidence then.

$$\frac{1}{2}\rho V^2 S\left[cC_{M,0} + ecC_{L,0} + ec\frac{\partial C_L}{\partial \alpha}(\alpha+\theta)\right] = K\theta$$

where $\partial C_L/\partial\alpha$ is the wing lift curve slope.

Re arranging give.

$$\theta\left[K - \frac{1}{2}\rho V^2 Sec\frac{\partial C_L}{\partial \alpha}\right] = \frac{1}{2}\rho V^2 Sc\left[C_{m,0} + eC_{L,0} + e\frac{\partial C_L}{\partial \alpha} * \alpha\right]$$

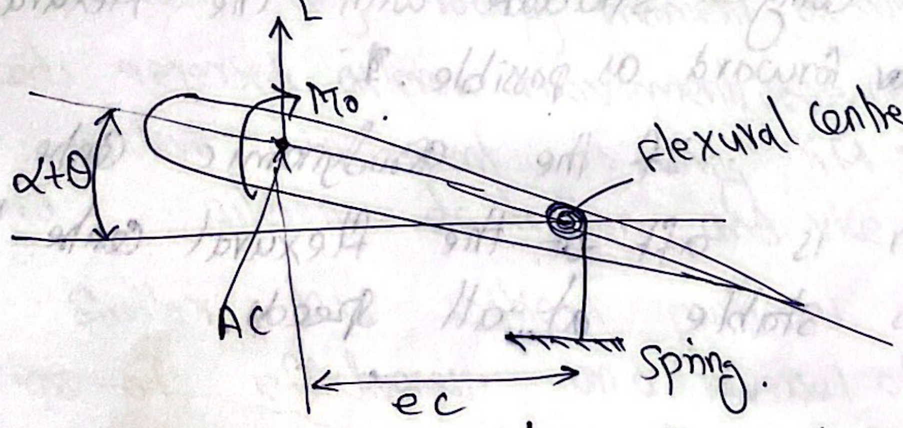

Fig: wing divergence speed (Two-dimensional case)

$$\theta = \frac{\frac{1}{2}\rho V^2 Sc\left[C_{M_0} + eC_{L0} + e(\partial C_L/\partial\alpha)\alpha\right]}{K - \frac{1}{2}\rho V^2 Sec\left(\frac{\partial C_L}{\partial \alpha}\right)}.$$

equation shows that divergence occurs when (θ = become infinite)

$$K = \frac{1}{2} \rho V^2 \sec \frac{\partial C_L}{\partial \alpha}.$$

∴ Then the divergence speed V_d is then.

$$V_d = \sqrt{\frac{2K}{\rho \sec \left(\frac{\partial C_L}{\partial \alpha}\right)}}$$

from the above equation that V_d may be increased either by stiffening the wing (increasing K) or by reducing the distance ec between the aero-dynamic and flexural centres.

The former approach involves weight and cost penalties so that designers usually prefer to design a wing structure with the flexural centre as far forward as possible.

If the aerodynamic centre coincides with or is aft of the flexural centre then the wing is stable at all speeds.