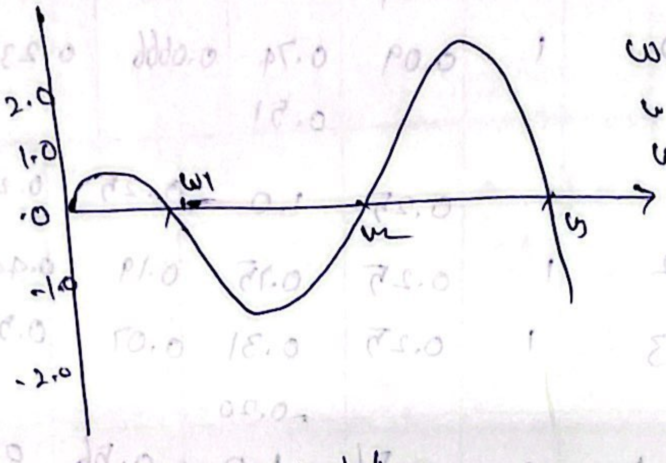
Similarly other deflections can be calculated and are directly put in the table for different assumed frequency. The results for frequency are obtained by drawing a curve between ω and displacement x.



$\omega_1 = 0.44$ rad/sec
$\omega_2 = 1.24$ rad/sec
$\omega_3 = 1.80$ rad/sec

$\therefore \omega_1 = 0.44$ rad/sec

$\omega_2 = 1.24$ rad/sec

$\omega_3 = 1.80$ rad/sec.

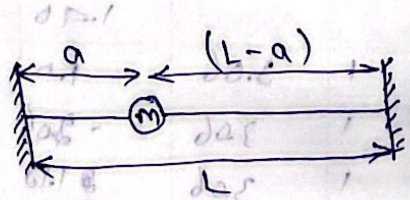
① Two ends of a string of length 'L' are rigidly fixed. It carries a lumped mass 'm' at a distance 'a' from left end. Find its natural frequency of transverse vibration of the string.

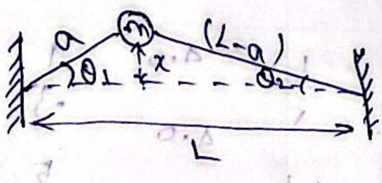
Given data:

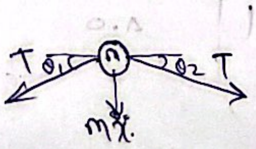
mass of the lumped mass $= m$

length of the string $= L$

To solve







$$\sin\theta_1 \approx \theta_1 = \frac{x}{a}$$

$$\sin\theta_2 \approx \theta_2 = \frac{x}{(L-a)}.$$

$$\theta_1 = \frac{x}{a}, \quad \theta_2 = \frac{x}{(L-a)}.$$

$$m\ddot{x} + T(\theta_1 - \theta_2) = 0.$$

$$m\ddot{x} + T\left[\frac{x}{a} - \frac{x}{(L-a)}\right] = 0$$

$$m\ddot{x} + Tx\left[\frac{1}{a} - \frac{1}{(L-a)}\right] = 0$$

$$m\ddot{x} + Tx\left[\frac{(L-a) - a}{a(L-a)}\right] = 0$$

$$m\ddot{x} + Tx\left[\frac{L-2a}{a(L-a)}\right] = 0.$$

Assume

$$x = e^{i\omega t}$$

$$\dot{x} = i\omega e^{i\omega t} = i\omega x$$

$$\ddot{x} = i^2\omega^2 e^{i\omega t}$$

$$= -\omega^2 e^{i\omega t}$$

$$\ddot{x} = -\omega^2 x$$

$$m(-\omega^2 e^{i\omega t}) + T(e^{i\omega t})\left[\frac{L-2a}{a(L-a)}\right] = 0.$$

$$\left\{-m\omega^2 + T\left[\frac{L-2a}{a(L-a)}\right]\right\} e^{i\omega t} = 0.$$

$$m\omega^2 = T\left[\frac{L-2a}{a(L-a)}\right]$$

$$\omega^2 = \frac{T}{m}\left[\frac{L-2a}{a(L-a)}\right]$$

$$\omega_{n} = \sqrt{\left(\frac{T}{m}\right)\left[\frac{L-2a}{a(L-a)}\right]} \;//.$$