ORTHOGONALITY PRINCIPLE:

The orthogonality Principle states that the principle modes of vibration for a mechanical system with more than one degree of freedom occurs along mutually perpendicular straight lines.

The orthogonality principle can be very useful for calculating the system's natural frequency.

For a two degree of freedom system orthogonality principle may be written as.

$$m_1 A_1 A_2 + m_2 B_1 B_2 = 0$$

A_1, A_2, B_1 and B_2 are the vibration amplitude of the coordinates for the first and second vibration modes.

For a three degree of freedom, orthogonality principle may be written as.

$$m_1 A_1 A_2 + m_2 B_1 B_2 + m_3 C_1 C_2 = 0$$

$$m_1 A_2 A_3 + m_2 B_2 B_3 + m_3 C_2 C_3 = 0$$

$$m_1 A_1 A_3 + m_2 B_1 B_3 + m_3 C_1 C_3 = 0.$$

EIGEN VALUES AND EIGEN VECTORS:-

from the matrix form of equation of motion,

$$m_1 \ddot{x}_1 + (K_1+K_2)x_1 - K_2 x_2 = 0$$

$$m_2 \ddot{x}_2 - K_1 x_1 + (K_1+K_2)x_2 - K_2 x_3 = 0$$

$$m_3 \ddot{x}_3 - K_3 x_2 + K_3 x_3 = 0.$$

$$[M]\{\ddot{x}\} + [K]\{x\} = 0.$$

$[M][m]^{-1} = [I]$ unit matrix

$$[I]\{\ddot{x}\} + [C]\{x\} = 0.$$

$[K][m]^{-1} = [C]$ dynamic matrix.

$$x = A \sin\omega t$$

$$\ddot{x} = -\omega^2 A \sin\omega t$$

$$= -\lambda A \sin\omega t$$

$$-\omega^2 [I]\{A\} + [C]\{A\} = 0$$

$\lambda = \omega^2 =$ eigen value.

$A =$ eigen vectors, $\begin{Bmatrix} A_1 \\ A_2 \\ A_3 \end{Bmatrix}$. $\begin{Bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{Bmatrix}$

$$\{\lambda_i[I] - [C]\}\{A\} = 0.$$

$$\boxed{\lambda_1 = 0.198\frac{k}{m}}$$

$$\lambda_1[I] - [C]$$

$$\left[\frac{0.198k}{m}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} - \frac{k}{m}\begin{bmatrix}2&-1&0\\-1&2&-1\\0&-1&1\end{bmatrix}\right]\begin{Bmatrix}A_1\\A_2\\A_3\end{Bmatrix}=0.$$

$$\frac{k}{m}\begin{bmatrix}-1.802&1&0\\1&-1.802&1\\0&1&-0.802\end{bmatrix}\begin{Bmatrix}A_1\\A_2\\A_3\end{Bmatrix}=0.$$

$$-1.802A_1 + A_2 = 0$$

$$A_1 - 1.802A_2 + A_3 = 0$$

$$A_2 - 0.802A_3 = 0.$$

mode Shape will be $= \begin{Bmatrix}A_1\\ \frac{A_2}{A_1}\\ \frac{A_3}{A_1}\end{Bmatrix}$.

$$\frac{A_2}{A_1} = 1.802$$

$$\frac{A_3}{A_1} = 2.247.$$

first mode shape will be

$$A_{i1} = A_1\begin{Bmatrix}1.0\\1.802\\2.247\end{Bmatrix},$$

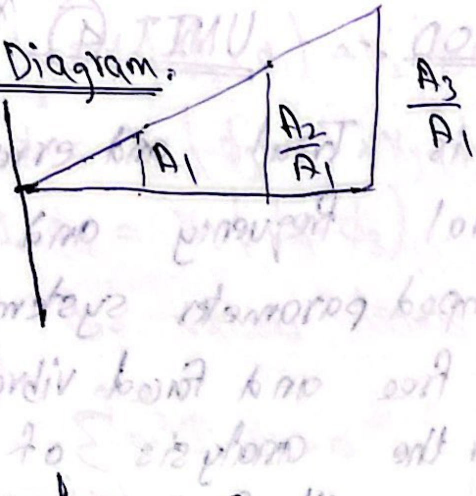
$$\boxed{\lambda_2 = 1.555\frac{k}{m}}$$

$$A_{2i} = A_2\begin{Bmatrix}1.0\\0.445\\-0.8019\end{Bmatrix}$$

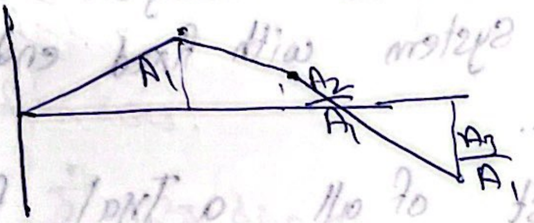
$$\boxed{\lambda_3 = 3.247\frac{k}{m}}$$

$$A_{3i} = A_3\begin{Bmatrix}1.0\\-1.247\\0.555\end{Bmatrix},$$

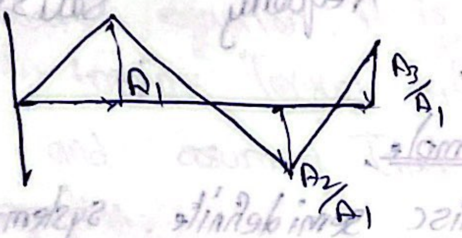
Mode shape Diagram:



First mode



Second mode



Third mode.

Result :-

$$\lambda_1 = 0.198\frac{K}{m}$$

$$\lambda_2 = 1.555\frac{K}{m}$$

$$\lambda_3 = 3.247\frac{K}{m}$$

$$A_{i1} = A_1 \begin{Bmatrix} 1.0 \\ 1.802 \\ 2.247 \end{Bmatrix}$$

$$A_{2i} = A_2 \begin{Bmatrix} 1.0 \\ 0.445 \\ -0.8019 \end{Bmatrix}$$

$$A_{3i} = A_3 \begin{Bmatrix} 1.0 \\ -1.247 \\ 0.555 \end{Bmatrix}$$