MATRIX METHOD:

This method is very convenient way to solve
the equation of motion. The lowest natural frequency of the
system can be determined very quickly by this method.
matrix method is very important to analysis as it is the basis
of many computer solutions.

$$[m]\{\ddot{x}\} + [K]\{x\} = 0.$$

$$\{\ddot{x}\} + [m]^{-1}[K]\{x\} = 0$$

$$\{\ddot{x}\} + [C]\{x\} = 0.$$

$[m]^{-1} = \frac{adj\, m}{|m|}$

$[C]$ = Dynamic matrix.

$$\{\ddot{x}\} = -\omega^2\{x\}$$

$$[C]\{x\} - \omega^2\{x\} = 0. \qquad \omega^2 = \lambda,$$

$$[C]\{x\} - \lambda\{x\} = 0$$

$$[\lambda I - C]\{x\} = 0.$$

Solution of

$$|\lambda I - C| = 0.$$

I = identity matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

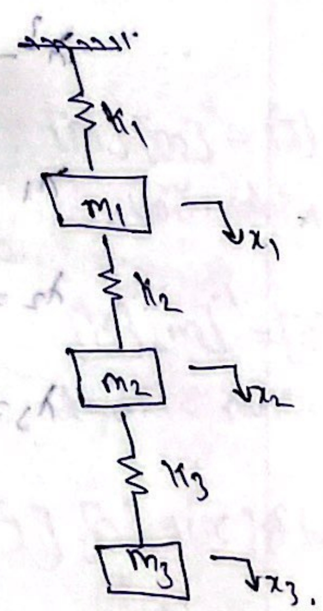
solving.

$$\cancel{\lambda^3} - \cancel{\lambda^2} \cancel{\lambda} + \lambda \cancel{\lambda^2} \cdots = 0.$$

$$m_1\ddot{x}_1 = -k_1x_1 - k_2(x_1 - x_2)$$

$$m_2\ddot{x}_2 = k_2(x_1 - x_2) - k_3(x_2 - x_3)$$

$$m_3\ddot{x}_3 = k_3(x_2 - x_3)$$



$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = 0.$$

$m_1 = m_2 = m_3 = m$

$k_1 = k_2 = k_3 = K.$

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = 0.$$

$$|m| = m^3$$

$$m^{-1} = \frac{\text{adj}\, m}{|m|} = \frac{1}{m^3}\begin{bmatrix} m^2 & 0 & 0 \\ 0 & m^2 & 0 \\ 0 & 0 & m^2 \end{bmatrix}$$

$$[C] = [m]^{-1}[K]$$

$$= \frac{1}{m}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix}$$

$$[C] = \frac{k}{m}\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$|\lambda I - C| = 0$$

$$\begin{vmatrix} \lambda - 2\frac{k}{m} & \frac{k}{m} & \\ \frac{k}{m} & \lambda - 2\frac{k}{m} & \frac{k}{m} \\ 0 & \frac{k}{m} & \lambda - \frac{k}{m} \end{vmatrix} = 0$$

$$\lambda^3 - 5\lambda^2\frac{k}{m} + 6\lambda\frac{k^2}{m^2} - \frac{k^3}{m^3} = 0.$$

$\lambda_1 = \omega_1^2 = 0.198\, k/m$; $\omega_1 = 0.44\sqrt{\frac{k}{m}}$ rad/sec

$\lambda_2 = \omega_2^2 = 1.555\frac{k}{m}$; $\omega_2 = 1.24\sqrt{\frac{k}{m}}$ rad/sec

$\lambda_3 = \omega_3^2 = 3.247\frac{k}{m}$; $\omega_3 = 1.80\sqrt{\frac{k}{m}}$ rad/sec.