SEVERAL DEGREES OF FREEDOM SYSTEM

The systems having more than one degree of freedom are known as several or multidegrees of freedom systems. We have already discussed two degrees of freedom systems. A system must have as many equations of motion and as many natural frequencies as the number of degrees of freedom. As the number of degrees of freedom increases, it becomes very tedious solving the equation of motion and to determine the natural frequencies and mode shapes. So we need approximate methods, like Rayleigh's method, Holzer's method, Dunkerley's method, Stodla method, matrix method, Rayleigh - Ritz method.

Influence Coefficient:-

The equations of motion of several degrees of freedom system can be expressed in terms of influence co-efficients. The influence coefficient a_{ij} is defined as the static deflection at point i because of unit load acting at point j. Similarly a_{ji} the deflection at point j due to unit load at point i.

According to maxwell's Reciprocal Theorem.

$$a_{ij} = a_{ji}$$

$$a_{12} = a_{21}$$

$$a_{31} = a_{13}.$$

If a system made of several points is acted by several forces $F_1, F_2, F_3 \ldots F_n$ causing respective deflection $x_1, x_2, x_3 \ldots x_n$ then

$$x_1 = a_{11}F_1 + a_{12}F_2 + a_{13}F_3 + \ldots a_{1n}F_n$$

$$x_2 = a_{12}F_1 + a_{22}F_2 + a_{23}F_3 + \ldots a_{2n}F_n.$$

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ \vdots \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & - & a_{nn} \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{Bmatrix}.$$

$\{x\} = [a]\{F\}$ $\quad$ a = Flexibility matrix

$$\begin{Bmatrix} F_1 \\ f_2 \\ \vdots \\ F_n \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & \dots & k_{1n} \\ k_{21} & \dots & \dots & \vdots \\ k_{n1} & \dots & \dots & k_{nn} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}.$$

$$\{F\} = [K]\{x\}.$$

$$\{x\} = [a][K]\{x\} \qquad [a][K] = [I].$$

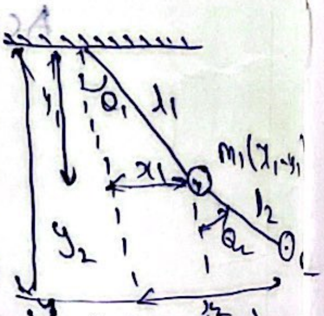
$$[a] = \frac{1}{[K]}.$$

GENERALIZED COORINATES.

*The configuration of a system is completely specified by certain independent parameters or coordinates which are known as generalized coordinates.

*These parameters specify the motion of a system completely. If a system has n degrees of freedom it will have n generalized coordinates. The generalized coordinates are generally denoted $q_1, q_2, \dots q_n$.

we consider double pendulum.



$$x_1 = l_1 \sin\theta_1, \quad x_2 = l_1 \sin\theta_1 + l_2 \sin\theta_2$$

$$y_1 = l_1 \cos\theta_1, \quad y_2 = l_1 \cos\theta_1 + l_2 \cos\theta_2.$$

Here x_1, y_1, x_2 and y_2 are not the generalized coordinates. Only θ_1 and θ_2 are the two independent coordinates which specify the system completely. These are the generalized coordinates

$$\theta_1 = q_1 \quad \text{and} \quad \theta_2 = q_2.$$

Thus generalized coordinates are set of independent coordinates equal in number of the degree of the system.