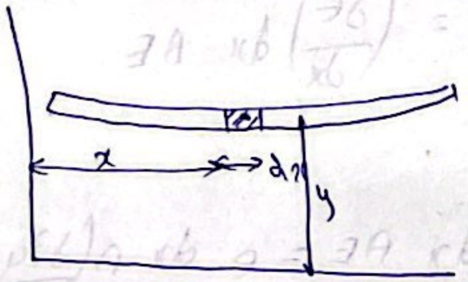
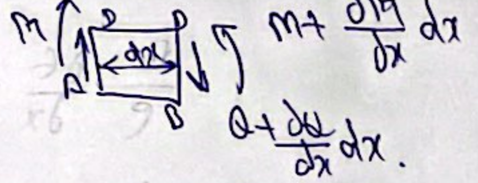
TRANSVERSE VIBRATION OF BEAMS:

If the cross-sectional dimensionals of beam are small compared to its length, The system is knows as Euler - Bernoulli beam. Only the thin beams are treated under it.





The differential equation for transverse vibration of thin uniform beam is obtained with the help of strength of materials. The beam has cross section area A, flexural rigidity EI and density of material e. The element dx of beam is subjected to shear force Q and bending moment M.

While deriving mathematical expression for transverse vibrations it is assumed that there are no axial forces acting on the beam and effect of shear deflection are neglected. The deformation of beam is assumed due to moment and shear force.

Net forces acting on the element.

$$-Q - \left(Q + \frac{\partial Q}{\partial x} dx\right) = dm \,.\, \text{acceleration}.$$

$$-\frac{\partial Q}{\partial x} dx = (\rho A dx)\left(\frac{\partial^2 y}{\partial t^2}\right)$$

$$\frac{\partial Q}{\partial x} + \rho A \frac{\partial^2 y}{\partial t^2} = 0$$

Considering the moments about A, we get.

$$M - \left(M + \frac{\partial M}{\partial x} dx\right) + \left(Q + \frac{\partial Q}{\partial x} dx\right) dx = 0$$

$$-\frac{\partial M}{\partial x} + Q + \frac{\partial Q}{\partial x} dx = 0.$$

So, $Q = \frac{\partial M}{\partial x}$ higher order derivatives are neglected

$\frac{\partial Q}{\partial x} dx = 0,$

$$\frac{\partial Q}{\partial x} = \frac{\partial^2 M}{\partial x^2}$$

$$\frac{\partial^2 M}{\partial x^2} = \rho A \frac{\partial^2 y}{\partial t^2}$$

$$\frac{M}{I} = \frac{E}{R}$$

from beam theory

$$M = -EI \frac{\partial^2 y}{\partial x^2}$$

$$\frac{\partial^2 M}{\partial x^2} = -EI \frac{\partial^4 y}{\partial x^4}.$$

$$EI \frac{\partial^4 y}{\partial x^4} + \rho A \frac{\partial^2 y}{\partial t^2} = 0$$

$$\frac{\partial^4 y}{\partial x^4} + \left(\frac{\rho A}{EI}\right) \frac{\partial^2 y}{\partial t^2} = 0.$$

This is the general equation for transverse, vibration which is different from wave equation.