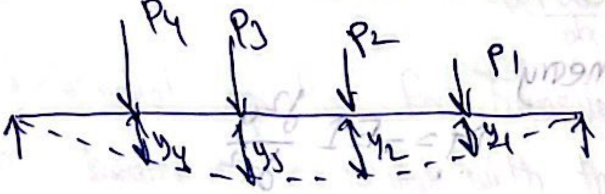
RAYEIGH METHOD.

This is the energy method to find the frequency. This method is used to find the natural frequency of the system when transverse point loads are acting on the beam or shaft. Good estimate of fundamental frequency can be made by assume the suitable deflection curve for the fundamental mode. The maximum kinetic energy is equated to maximum potential energy of the system to determine the natural frequency.

Let us consider a shaft AB of negligible weight. several point loads P_1, P_2, P_3.



The maximum potential energy of the system

$$P.E = \frac{1}{2} P_1 y_1 + \frac{1}{2} P_2 y_2 + \frac{1}{2} P_3 y_3 + \frac{1}{2} P_4 y_4$$

$$= \frac{1}{2} \Sigma P_y$$

The maximum kinetic energy of the system can be written.

$$K.E = \frac{1}{2g} P_1 (\omega y_1)^2 + \frac{1}{2g} P_2 (\omega y_2)^2 + \frac{1}{2g} P_3 (\omega y_3)^2 + \frac{1}{2g} P_4 (\omega y_4)^2$$

$$= \frac{\omega^2}{2g} \Sigma P_y{}^2$$

ω = natural frequency.

Equating the maximum kinetic energy to maximum potential energy.

$$\frac{\omega^2}{2g}\sum P y^2 = \frac{1}{2}\sum P y$$

$$\omega = \sqrt{\frac{g\sum P y}{\sum P y^2}}$$

The above equation can be written in a more generalised way by including the distributed mass of the beams. If m is the mass of the beam per unit length and y is assumed deflection curve, the maximum potential energy of beam of length l

$$P.E = \frac{1}{2}\int_0^l M\, d\theta$$

M - bending moment

$d\theta$ = Change in slope over a distance dx

<u>from beam theory</u>

$$\frac{M}{I} = \frac{E}{R}$$, R - radius of curvature

EI - Flexural Rigidity.

$$\frac{1}{R} = \frac{d\theta}{dx} = \frac{d^2y}{dx^2} \Rightarrow d\theta = \left(\frac{d^2y}{dx^2}\right) dx.$$

$$M = \frac{EI}{R} = EI\frac{d^2y}{dx^2}\, dx$$

$$P.E = \frac{1}{2}\int_0^l EI\left(\frac{d^2y}{dx^2}\right)^2 dx$$

The maximum Kinetic energy of the system

$$K.E = \frac{1}{2}\int_0^l m(\omega y)^2\, dx.$$

Equating maximum kinetic energy to maximum potential energy.

$$\frac{1}{2}\int_0^l m\,(\omega y)^2\,dx = \frac{1}{2}\int_0^l EI\left(\frac{d^2y}{dx^2}\right)^2 dx$$

$$\omega^2 = \frac{\int_0^l EI\left(\frac{d^2y}{dx^2}\right)^2 dx}{\int_0^l m y^2\,dx}$$

$$\therefore \omega^2 = \frac{EI}{m}\,\frac{\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx}{\int_0^l y^2\,dx}.$$