LAGRANGE'S EQUATIONS

The equations of motion of a vibrating system are written in term of generalised coordinates by making use of Lagrange's equations. Generalised coordinates are independent parameters which specify the system completely. If energy expressions are available, the equations of motion can be obtained with help of Lagrange's equation. The general form of this equation in terms of generalised coordinates is written as.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_j}\right) - \frac{\partial T}{\partial x_j} + \frac{\partial V}{\partial x_j} = Q_j$$

T = Total kinetic energy of the system

V = Total potential energy of the system.

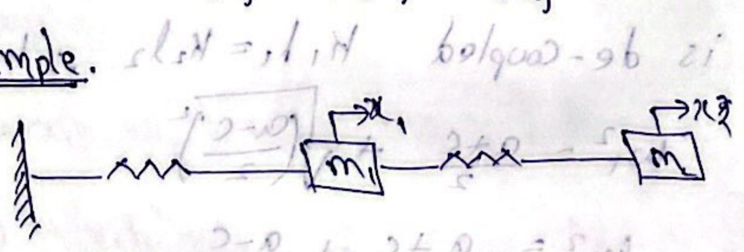
$j = 1, 2, 3, \ldots n$

Q_j = generalised external force.

For a conservative system generalised force Q_j acting on the system is zero, so equation for such a system can be written as.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_j}\right) - \frac{\partial T}{\partial x_j} + \frac{\partial V}{\partial x_j} = 0.$$

For example.



$$T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2$$

$$V = \frac{1}{2} K_1 x_1^2 + \frac{1}{2} K_2 (x_2 - x_1)^2$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1$$

$$\frac{\partial T}{\partial x_1} = 0.$$

$$\frac{\partial V}{\partial x_1} = K_1 x_1 - K_2 (x_2 - x_1)$$

$$m_1 \ddot{x}_1 + K_1 x_1 - K_2 (x_2 - x_1) = 0.$$

for second equation.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2$$

$$\frac{\partial T}{\partial x_2} = 0$$

$$\frac{\partial V}{\partial x_2} = K_2 (x_2 - x_1)$$

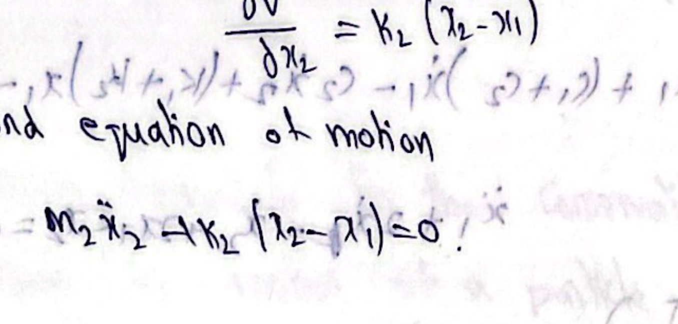
Second equation of motion

$$m_2 \ddot{x}_2 + K_2 (x_2 - x_1) = 0.$$

__Derive the equation of motion for the system by using Lagrange's Equation.__

$m_1 = m_2 = 1$, $c_1 = c_2 = c_3 = 1$

$K_1 = K_2 = K_3 = 1$



Kinetic energy.

$$K.E = \frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 = T$$

potential Energy

$$P.E = \frac{1}{2} K_1 x_1^2 + \frac{1}{2} K_2 (x_1 - x_2)^2 + \frac{1}{2} K_3 x_2^2 = V.$$

dissipation energy.

$$D.E = \frac{1}{2} C_1 \dot{x}_1^2 + \frac{1}{2} c_2 (\dot{x}_1 - \dot{x}_2)^2 + \frac{1}{2} c_3 \dot{x}_2^2 = D.$$

Lagrange's equation.

$$\frac{d}{dt} \frac{\partial}{\partial \dot{x}_j} (K.E) - \frac{\partial (K.E)}{\partial x_j} + \frac{\partial (P.E)}{\partial x_j} + \frac{\partial (D.E)}{\partial \dot{x}_j} = 0$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1$$

$$\frac{\partial T}{\partial x_1} = 0.$$

$$\frac{\partial V}{\partial x_1} = K_1 x_1 + K_2(x_1 - x_2)$$

$$= (K_1 + K_2)x_1 - K_2 x_2$$

$$\frac{\partial D}{\partial \dot{x}_1} = c_1 \dot{x}_1 + c_2(\dot{x}_1 - \dot{x}_2)$$

$$= (c_1 + c_2)\dot{x}_1 - c_2 \dot{x}_2.$$

$$m_1 \ddot{x}_1 + (c_1 + c_2)\dot{x}_1 - c_2 \dot{x}_2 + (K_1 + K_2)x_1 - K_2 x_2 = 0.$$

$$\ddot{x}_1 + 2\dot{x}_1 - \dot{x}_2 + 2x_1 - x_2 = 0.$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2$$

$$\frac{\partial T}{\partial x_2} = 0$$

$$\frac{\partial V}{\partial x_2} = k_3 x_2 + \frac{1}{2} k_2 (x_1 - x_2) \times -1$$

$$= x_2 k_3 + \cancel{\tfrac{1}{2}} k_2 (x_2 - x_1)$$

$$\frac{\partial (D.E)}{\partial \dot{x}_2} = c_3 \dot{x}_2 + \cancel{\tfrac{1}{2}} k_2 (\dot{x}_2 - \dot{x}_1)$$

$$m_2 \ddot{x}_2 + x_2(c_3 + c_2) - c_2 \dot{x}_2 + x_2 k_3 + k_2(x_2 - x_1) = 0$$

$$m_2 \ddot{x}_2 + \dot{x}_2(c_3 + c_2) + x_2(k_3 + k_2) - c_2 \dot{x}_1 - k_2 x_1 = 0$$

$$\ddot{x}_2 + 2\dot{x}_2 + 2x_2 - \dot{x}_1 - x_1 = 0$$