equation of motion.

$$m\ddot{x} = -K_2(x - l_2\theta) - K_1(x + l_1\theta)$$

$$I\ddot{\theta} = K_2(x - l_2\theta)l_2 - K_1(x + l_1\theta)l_1$$

$$m\ddot{x} + (K_1 + K_2)x - (K_2 l_2 - K_1 l_1)\theta = 0.$$

$$I\ddot{\theta} - (K_2 l_2 - K_1 l_1)x + (K_2 l_2^2 + K_1 l_1^2)\theta = 0$$

Above both equation have $x$ and $\theta$ terms, so they are the Coupled equation. the system has rotary as well as translatory motion.

if $K_1 l_1 = K_2 l_2$

$$m\ddot{x} + (K_1 + K_2)x = 0$$

$$I\ddot{\theta} + (K_2 l_2^2 + K_1 l_1^2)\theta = 0.$$

It can be seen from the above equation that translatory and angular motion can exist independently. These are called uncoupled differential equations. This is called dynamic coupling.

If the coupling term $(K_1 l_1 - K_2 l_2) \neq 0$ is non zero the coupling so formed is known as static or elastic coupling.

$$m\ddot{x} + x(K_1 + K_2) - (l_2 K_2 - K_1 l_1)\theta = 0$$

$$I\ddot{\theta} + (K_1 l_1^2 + K_2 l_2^2)\theta - (K_2 l_2 - K_1 l_1)x = 0.$$

$$\frac{K_1 + K_2}{m} = a_1 \; ; \; \frac{l_1 K_1 - K_2 l_2}{m} = b$$

$$\frac{K_1 l_1^2 + K_2 l_2^2}{I} = c$$

$(I = mr^2)$

$$\ddot{x} + ax = b\theta$$

$$\ddot{\theta} + c\theta = \frac{b}{r^2}x$$

$$x = X\sin\omega t \; ; \; \dot{x} = \omega X\cos\omega t \; ; \; \ddot{x} = -\omega^2 X\sin\omega t$$

$$\theta = \beta\sin\omega t \; ; \; \dot{\theta} = +\omega\beta\cos\omega t \; ; \; \ddot{\theta} = -\omega^2\beta\sin\omega t$$
$$= -\omega^2\theta.$$

$$-\omega^2 X\sin\omega t + a\,X\sin\omega t = b\,\beta\sin\omega t$$
$$-\omega^2 X + aX = b\beta$$

$$\frac{X}{\beta} = \frac{b}{(-\omega^2 + a)}$$

$$-\omega^2\beta\sin\omega t + c\,\beta\sin\omega t = \frac{b}{r^2}X\sin\omega t.$$

$$-\omega^2\beta + c\beta = \frac{b}{r^2}X$$

$$\frac{X}{\beta} = \frac{(-\omega^2 + c)}{b}r^2$$

$$\left(\frac{b}{-\omega^2 + a}\right) = \left(\frac{-\omega^2 + c}{b}\right)r^2$$

$$b^2 = (-\omega^2 + a)(-\omega^2 + c)r^2$$

$$\omega^4 - \omega^2(c + a) + ac - \frac{b^2}{r^2} = 0.$$

$$\omega_1^2 = \frac{1}{2}(a + c) - \sqrt{\frac{1}{4}(c - a)^2 + \frac{b^2}{r^2}}$$

$$\omega_2^2 = \frac{1}{2}(a + c) + \sqrt{\frac{1}{4}(c - a)^2 + \frac{b^2}{r^2}}$$

When it is de-Coupled $K_1 l_1 = K_2 l_2 \;\therefore b = 0$

$$\omega_1^2 = \frac{a + c}{2} + \sqrt{\left(\frac{a - c}{2}\right)^2}$$

$$\omega_1^2 = \frac{a + c}{2} + \frac{a - c}{2}$$

$$\omega_1^2 = a \qquad \therefore a = \frac{K_1 + K_2}{m}$$

$$\omega_2^2 = \left(\frac{a + c}{2}\right) - \frac{a - c}{2}$$

$$\omega_2^2 = c \qquad c = \frac{K_1 l_1^2 + K_2 l_2^2}{I}$$

$$I = mr^2.$$