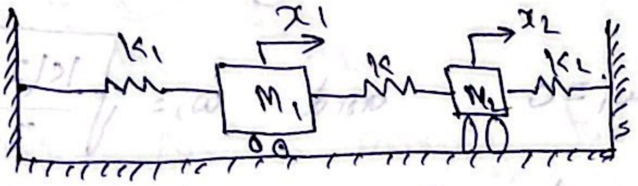
## Vibrations of Undamped Two Degrees of Freedom Systems:-




Two degrees of freedom system.

The two masses $m_1$ and $m_2$ are defined by their position $x_1$ and $x_2$ respectively at any Time t.

equation of motion for the two masses.

$$m_1\ddot{x}_1 + (K_1+K)x - K x_2 = 0$$

$$m_2\ddot{x}_2 + (K+K_2)x_2 - Kx_1 = 0.$$

Assumed

$$x_1 = A_1 \sin(\omega t+\phi)$$
$$x_2 = A_2 \sin(\omega t+\phi)$$

$$-m_1 A_1 \omega^2 \sin(\omega t+\phi) + (K_1+K) A_1 \sin(\omega t+\phi) - K A_2 \sin(\omega t+\phi) = 0,$$

$$A_1(K+K_1-m_1\omega^2) - A_2 K = 0.$$

$$-A_1 K + A_2 (K_2+K-m_2\omega^2) = 0.$$

There are homogeneous linear algebraic equation in $A_1$ and $A_2$. The solution is obtained by equating to zero the determinant of the coefficient of $A_1$ and $A_2$.

$$\begin{vmatrix} k_1+k-m_1\omega^2 & -k \\ -k & k_2+k-m_2\omega^2 \end{vmatrix} = 0.$$

$$(k_1+k-m_1\omega^2)(k_2+k-m_2\omega^2) - k^2 = 0.$$

$$\omega^4 - \left[\frac{k+k_2}{m_2} + \frac{k_1+k}{m_1}\right]\omega^2 + \frac{k_1k_2 + k_1k + kk_2}{m_1m_2} = 0.$$

This is the frequency equation and is a quadratic in $\omega^2$ and gives two values of $\omega^2$.

If we assume $k_1 = k_2 = k$

$m_1 = m_2 = m$

$$\omega^4 - \left[\frac{2k}{m} + \frac{2k}{m}\right]\omega^2 + \frac{3k^2}{m^2} = 0.$$

$$\omega^4 - \frac{4k}{m}\omega^2 + \frac{3k^2}{m^2} = 0$$

$$\omega^2 = \frac{\frac{4k}{m} \pm \sqrt{\frac{16k^2}{m^2} - \frac{12k^2}{m^2}}}{2}$$

$$\omega_1 = \sqrt{\frac{k}{m}} \qquad \omega_2 = \sqrt{\frac{3k}{m}}$$

$\omega_1$, $\omega_2$ First and second modes.

Amplitude ratios are

$$\omega_1 \Rightarrow \frac{A_1}{A_2} = \frac{k}{k+k_1-m_1\omega^2} = \frac{k}{2k-m\omega^2}$$

$$\omega_2 \Rightarrow \frac{A_1}{A_2} = \frac{k_2+k-m_2\omega^2}{k} = \frac{2k-m\omega^2}{k}.$$

$$\left(\frac{A_1}{A_2}\right)_1 = \frac{k}{2k - \frac{k}{m} \cdot m} = 1 \qquad \omega = \sqrt{\frac{k}{m}}$$

$$\left(\frac{A_1}{A_2}\right)_2 = \frac{2k - m \cdot \frac{3k}{m}}{k} = -1 \qquad \omega = \sqrt{\frac{3k}{m}}$$

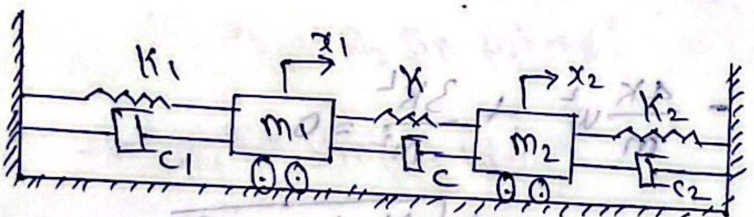
$$\therefore \omega_1 = 2\pi f_1 \qquad \omega_2 = 2\pi f_2$$

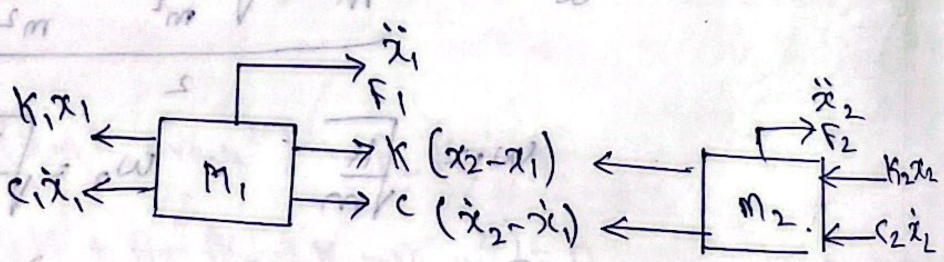
$$f_1 = \frac{\omega_1}{2\pi} \qquad f_2 = \frac{\omega_2}{2\pi}$$

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}\ \text{Hz} \qquad f_2 = \frac{1}{2\pi}\sqrt{\frac{3k}{m}}\ \text{Hz}$$

## (matrix) FORCED VIBRATIONS:-

A Viscouly damped two degrees of freedom system. The system having two masses $m_1$ and $m_2$ is put to excitation force $F_1$ and $F_2$.




$$m_1\ddot{x}_1 + K_1 x_1 + K(x_1 - x_2) + c_1\dot{x}_1 + c(\dot{x}_1 - \dot{x}_2) = F_1(t)$$

$$m_2\ddot{x}_2 + K_2 x_2 + K(x_2 - x_1) + c_2\dot{x}_2 + c(\dot{x}_2 - \dot{x}_1) = F_2(t)$$

$$m_1\ddot{x}_1 + (c_1 + c)\dot{x}_1 + (K_1 + K)x_1 - c\dot{x}_2 - Kx_2 = F_1(t)$$

$$m_2\ddot{x}_2 + (c_2 + c)\dot{x}_2 + (K_2 + K)x_2 - c\dot{x}_1 - Kx_1 = F_2(t)$$