Unit - 3

Two Degrees of Freedom system:

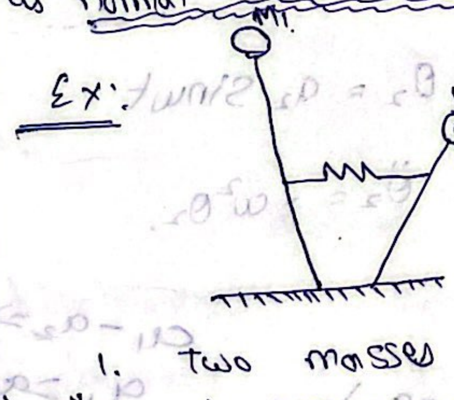
Two co-ordinates independently to describe its motion completely is called a two degree of freedom system. In such a system there are two masses which will have two natural frequencies. so the system will be having two equation of motion which may be treated as coupled differential equation.

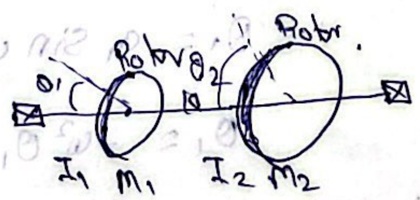
The system at its lowest or first natural frequency is called its first mode. at its next second higher is called the second mode and so on.

If the two masses vibrate at the same frequency and in phase it is called a principle mode of vibration.

If at the principal mode of vibration, the amplitude of one of the masses is unity it is known as normal mode of vibration

Ex:





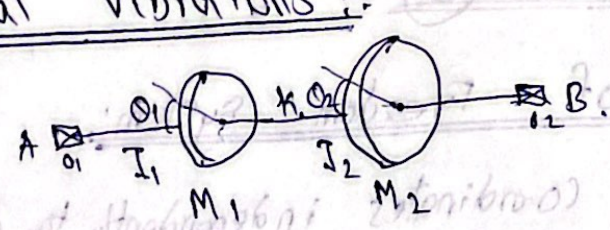
1. Two masses of a simple pendulum are coupled together by means of a spring.

2. similarly a shaft of torsional stiffness k is having two rotors which can have angular displacement θ_1 and θ_2 independent of each other. Thus it is a two degree of freedom system.

$I_1 - I_2$ - moment of inertia.

$\theta_1 - \theta_2$ - angular rotation

Torsional Vibrations :-



Shaft AB is carrying two Rotors having moment of inerties as I_1 and I_2. Let θ_1 and θ_2 be the angular displacement of the rotors at any instant from the mean position the equation of motion for each rotor can be written as

$$I_1 \frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta_2) = 0 \quad \text{I}$$

$$I_2 \frac{d^2\theta_2}{dt^2} + K(\theta_2 - \theta_1) = 0 \quad \text{II}$$

Assumed the solution

$$\theta_1 = a_1 \sin \omega t \qquad \theta_2 = a_2 \sin \omega t$$

$$\ddot{\theta}_1 = -\omega^2 \theta_1 \qquad \ddot{\theta}_2 = -\omega^2 \theta_2.$$

Substituting these values.

$$-\omega^2 I_1 a_1 + K(a_1 - a_2) = 0.$$

$$-\omega^2 I_2 a_2 + K(a_2 - a_1) = 0.$$

$a_1 - a_2 = 0$

$a_1 = a_2$

$\frac{a_1}{a_2} = 1$

equating the determinant of the above equation.

$$(-\omega^2 I_1 + K)(-\omega^2 I_2 + K) - K^2 = 0$$

$$\omega^4 I_1 I_2 - \omega^2 I_1 K - \omega^2 K I_2 + \cancel{K^2} - \cancel{K^2} = 0$$

$$\omega^2\left[\omega^2 I_1 I_2 - I_1 K - I_2 K\right] = 0$$

$$\omega^2\left[\omega^2 - \frac{K}{I_1 I_2}\left[I_1+I_2\right]\right] = 0$$

$$\therefore \quad \omega_1 = 0 \quad \text{and} \quad \omega_2 = \sqrt{\frac{K(I_1+I_2)}{I_1 I_2}}$$

putting the value of ω_1 in equation

$$\frac{a_1}{a_2} = 1.$$

putting the value of ω_2 in equation

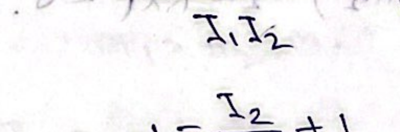
$$\frac{a_1}{a_2} = \frac{-\omega^2 I_2 + K}{K} = \frac{-\omega^2 I_2}{K} + 1$$

$$= \frac{-K(I_1+I_2)}{I_1 I_2} \cdot \frac{I_2}{K} + 1$$

$$= -1 - \frac{I_2}{I_1} + 1$$

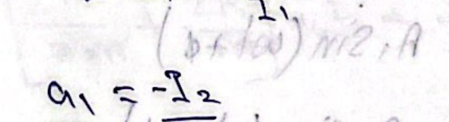
$$\frac{a_1}{a_2} = \frac{-I_2}{I_1}.$$

The section of the shaft where the angular displacement is zero, is known as node. The angular displacement of the rotor are inversely proportional to their inertias $\therefore \left(\frac{a_1}{a_2} = -\frac{I_2}{I_1}\right)$. The first and second mode shapes.



(Torsional vibration)

$\omega_n = 0$



→ First mode shape

$$\omega_n = \sqrt{\frac{K(I_1+I_2)}{I_1 I_2}}$$

→ Second mode shape.