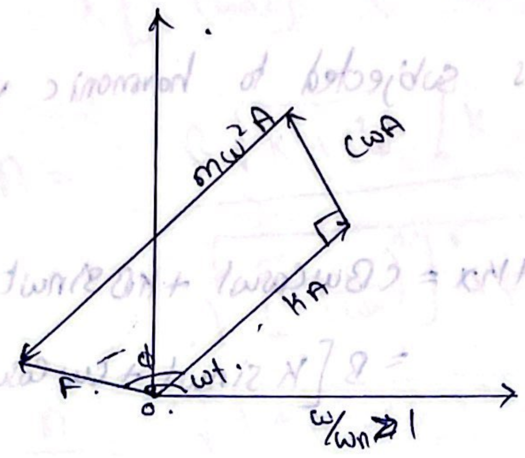
Case 3 $\omega/\omega_n \gg 1$.

At very high frequencies ω inertia force increase very rapidly and its magnitude is very large. Damping and spring forces are small in magnitude when the value of ω/ω_n is very high. Phase angle ϕ is very close to $180°$.



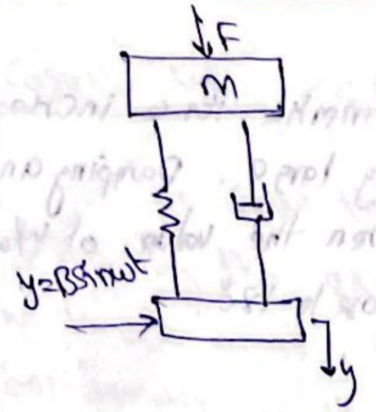
SUPPORT MOTION:

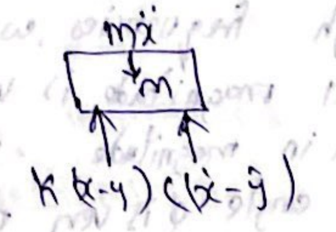
Locomotives or vehicles the wheels act as base or supports for the system. The wheels can move vertically up and down on the road surface during the motion of the vehicle. At the same time, there is relative motion between the wheels and the chassis.

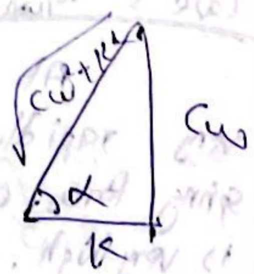
Absolute motion:

Absolute motion of a mass means its motion with respect to the coordinate system attached to the earth. Absolute displacement of supports is $y = B\sin\omega t$ and the absolute displacement of the mass from its equilibrium position is x. The displacement of mass m relative to the support is z. The net elongation of the spring $(x-y)$ and relative motion between the two ends of the damper is $(\dot{x}-\dot{y})$.

The
$$z = x - y$$
$$\dot{z} = \dot{x} - \dot{y}$$
$$m\ddot{x} + c(\dot{x}-\dot{y}) + k(x-y) = 0$$
$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky.$$







The support is subjected to harmonic vibration.

$y = B\sin\omega t$.

$$m\ddot{x}+c\dot{x}+kx = cB\omega\cos\omega t + kB\sin\omega t$$

$$= B\left[k\sin\omega t + c\omega\cos\omega t\right]$$

$$= B\sqrt{k^2+c^2\omega^2}\left[\frac{k}{\sqrt{k^2+c^2\omega^2}}\sin\omega t + \frac{c\omega}{\sqrt{k^2+c^2\omega^2}}\cos\omega t\right]$$

$$= B\sqrt{k^2+c^2\omega^2}\left[\cos\alpha\sin\omega t + \sin\alpha\cos\omega t\right]$$

$$m\ddot{x}+c\dot{x}+kx = B\sqrt{k^2+c^2\omega^2}\sin(\omega t+\alpha)$$

$$\tan\alpha = \frac{c\omega}{k}$$

$$\alpha = \tan^{-1}\frac{c\omega}{k}$$

$$= \tan^{-1}\left(2\xi\frac{\omega}{\omega_n}\right)$$

Steady state solution gives the system behaviour during a sufficiently long time interval 't'.

Steady state solution can be written as

$$x = A\sin(\omega t+\alpha-\phi)$$

$$m\ddot{x}+c\dot{x}+kx = F\sin(\omega t+\alpha)$$

$$F = B\sqrt{K^2 + c^2\omega^2}.$$

$$A = \frac{F/K}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$A = \frac{B\sqrt{K^2 + c^2\omega^2}\,/K}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$\frac{A}{B} = \frac{\sqrt{1 + (2\xi\,\omega/\omega_n)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}.$$

ratio $\frac{A}{B}$ is called the displacement transmissibility. which is the ratio of the amplitude of the body to the amplitude of the support.

Relative motion:-

From the Absolute motion we have assumed that z is the relative displacement of the mass (m) with respect to the support:

$$z = x - y \quad , \dot{z} = \dot{x} - \dot{y} \quad , \ddot{z} = \ddot{x} - \ddot{y}$$

$$m\ddot{x} + c\dot{z} + Kz = 0 . \qquad \ddot{x} = \ddot{z} + \ddot{y}$$

$$m(\ddot{z} + \ddot{y}) + c\dot{z} + Kz = 0$$

$$m\ddot{z} + c\dot{z} + Kz = -m\ddot{y} = m\omega^2 B\sin\omega t .$$

$$z = Z\sin(\omega t - \alpha)$$

$$\frac{Z}{B} = \frac{(\omega/\omega_n)^2}{\sqrt{\left[1-(\omega/\omega_n)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$\alpha = \tan^{-1}\left[\frac{2\xi(\omega/\omega_n)}{1-(\omega/\omega_n)^2}\right]$$