

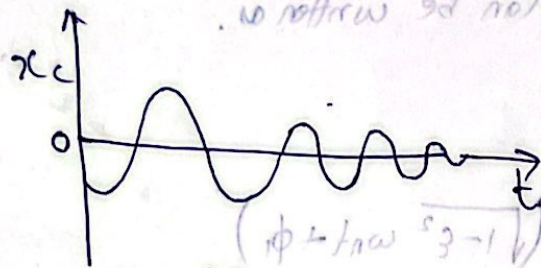
At resonance $\omega = \omega_n$ putting eqn (7)

$$\frac{A}{x_s} = \frac{1}{2\zeta}$$

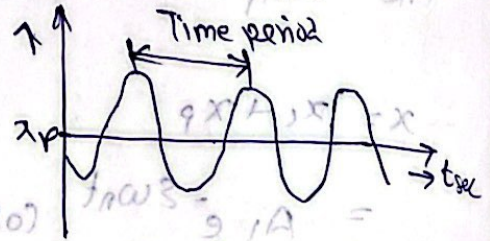
Total Response:-

The first part of eqn (9) vanishes with time while the second part remains into existence. The amplitude remains constant due to second part and it is called steady vibration. The vibration because of first part is called transient and it occurs at the damped natural frequency of the system.

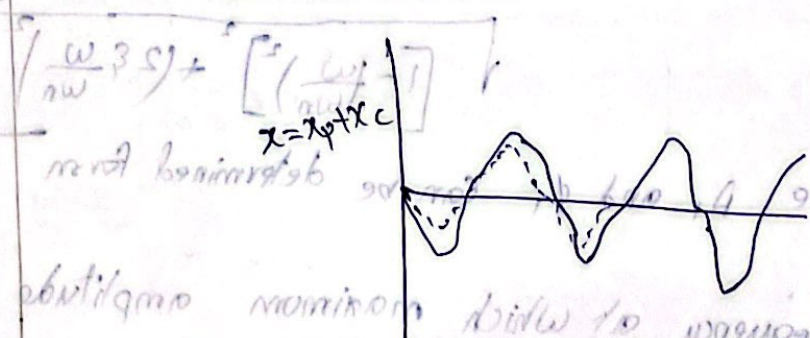
The complete solution of eqn (9) is the superposition of transient and steady vibration.



(a) Transient vibration.



(b) Steady state vibration.



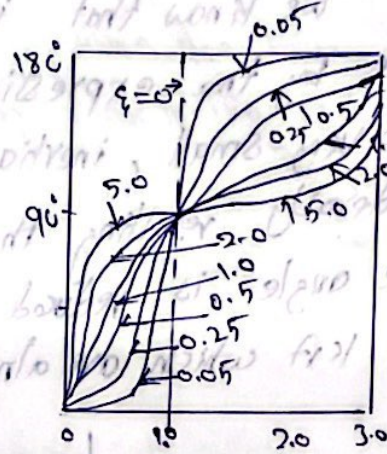
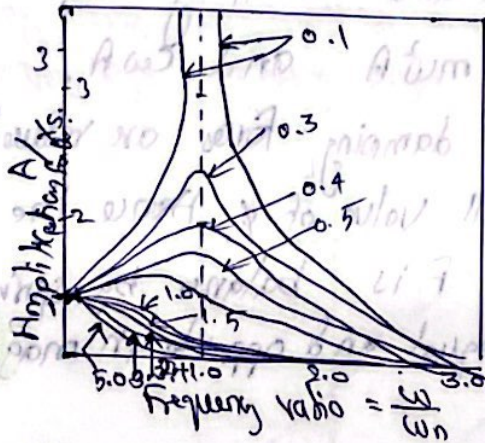
(c) Super position of transient and steady state.

Characteristic curves:-

The ratio ω/ω_n is called the frequency ratio where ω is the frequency of excitation. Similarly x/x_s is magnification factor or amplitude ratio.

A curve between magnification factor is known as frequency response curve.

Similarly a curve between phase angle and frequency ratio is known as phase - frequency response curve.



Following points are noted from these equations and fig.

1. At zero frequency magnification is unity and damping does not have any effects on it.

2. Damping reduces the magnification factor for all value of frequency.

3. The maximum value of amplitude occurs a little towards left of resonant frequency.

4. At resonant frequency the phase angle is 90° .

5. The phase angle increases for decreasing value of damping above resonance.

6. The amplitude of vibration is infinite at resonant frequency and zero damping factor.

7. The amplitude ratio is below unity for all values of damping which are more than 1.0.

8. The variation in phase angle is because of damping without damping it is either 180° or 0° .

Variation of frequency ratio (ω/ω_n)

There are three possibility.

(i) $\frac{\omega}{\omega_n} < 1$

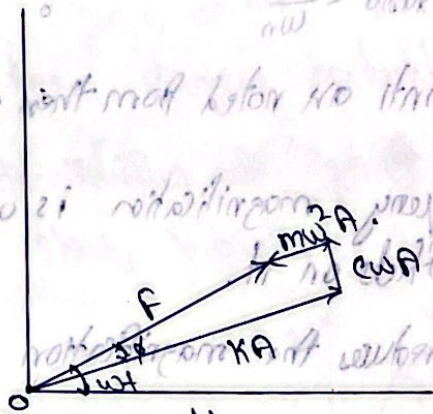
(ii) $\frac{\omega}{\omega_n} = 1$

(iii) $\frac{\omega}{\omega_n} \gg 1$

This is because of the variation of ω will affect the magnitude of various forces acting on the system.

Case (i) $\omega/\omega_n \ll 1$

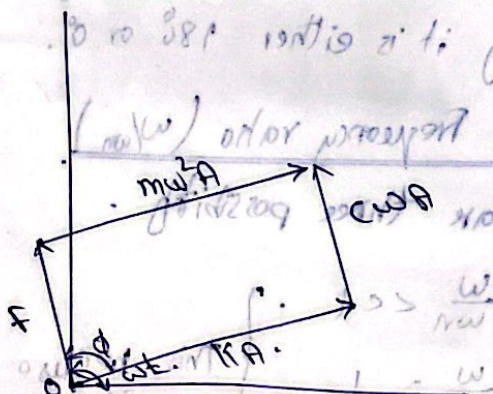
We know that inertia and damping forces are given by the expressions $m\omega^2 A$ and $c\omega A$. ω is very small, inertia and damping forces are reduced considerably resulting the small value of ϕ . Hence the phase angle is reduced and F is balance by spring force kA which are almost equal and opposite in magnitude.



(a) $\frac{\omega}{\omega_n} \leq 1$

Case (ii) $\frac{\omega}{\omega_n} = 1$

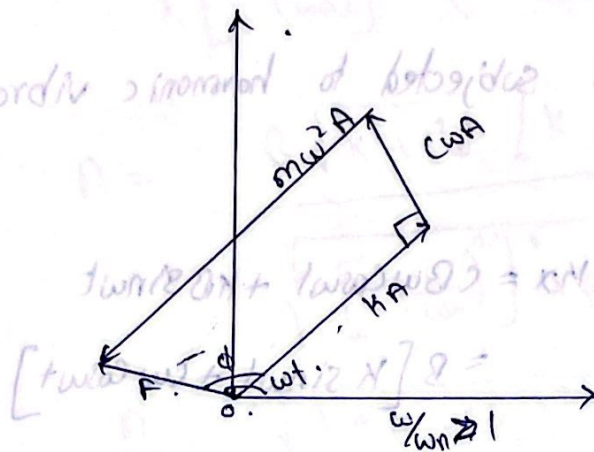
When the frequency of excitation ω increase and becomes equal to the natural frequency ω_n , resonance occurs. The phase angle becomes 90° . Inertia force and spring force are found to be equal and opposite. Excitation force balance the damping force i.e. $c\omega A = F$. Thus giving the amplitude of vibration at resonance.



(i) $\omega < \omega_n$
 (ii) $\omega = \omega_n$
 (iii) $\omega > \omega_n$

Case 3 $\omega/\omega_n \gg 1$.

At very high frequencies ω inertia force increase very rapidly and its magnitude is very large. Damping and Spring forces are small in magnitude. When the value of ω/ω_n is very high, phase angle ϕ is very close to 180° .



SUPPORT MOTION:

Locomotives or vehicles the wheels act as base or supports for the system. The wheels can move vertically up and down on the road surface during the motion of the vehicle.

At the same time there is relative motion between the wheels and the chassis.

Absolute motion:

Absolute motion of a mass means its motion with respect to the coordinate system attached to the earth. Absolute displacement of supports is $y = B \sin \omega t$ and the absolute displacement of the mass from its equilibrium position is x . The displacement of mass m relative to the support is z . The net elongation of the spring ($x-y$) and relative motion between the two ends of the damper is $(\dot{x}-\dot{y})$.

The $z = x - y$

$\dot{z} = \dot{x} - \dot{y}$

$m\ddot{x} + c(\dot{x}-\dot{y}) + k(x-y) = 0$

$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$