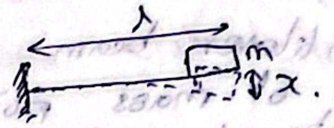
(11) Determine the natural frequency of the mass m placed at one end of a cantilever beam of negligible mass.



Deflection $= \frac{W l^3}{3EI}$.

stiffness $= \frac{\text{load}}{\text{deflection}}$.

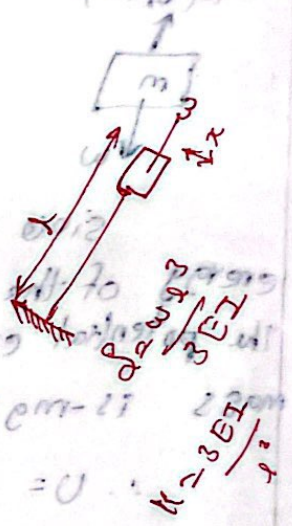
Solution:

The stiffness of beam is given as.

$$k = 3EI / l^3$$

where EI is the flexural rigidity of the beam

The general equation of motion for undamped free vibration is given as



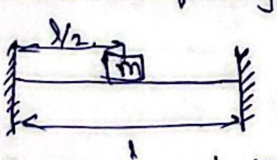
$$m\ddot{x} + kx = 0.$$

$$m\ddot{x} + \frac{3EI}{l^3} x = 0.$$

$$\omega_n = \sqrt{\frac{3EI}{l^3 m}} \text{ rad/sec}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{3EI}{m l^3}} \text{ Hz.}$$

(12) Find the natural frequency of the system.



solution:

The deflection at the centre of a bar fixed at both ends with load w at the centre.

deflection $= \frac{W l^3}{192EI}$. stiffness $= \frac{192EI}{l^3}$.

G.E:

$$m\ddot{x} + kx = 0.$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{192EI}{m l^3}} \text{ rad/sec}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{192EI}{m l^3}} \text{ Hz.}$$

(13) Determine the effect of the mass of the spring on the natural frequency of the system show in fig.

Solution

Let x be the displacement of mass m and so the velocity will be $\dot{x}$. The velocity of spring element at a distance y from the fixed end may be written as $\frac{\dot{x}y}{l}$ where l is the total length of spring

The Kinetic energy of spring element dy is written as,

$$\frac{1}{2}(\rho\, dy)\left(\frac{y}{l}\dot{x}\right)^2$$

where ρ being the mass of spring per unit length

$\therefore$ Total Kinetic energy of the system.

$$K.E = \frac{1}{2}m\dot{x}^2 + \int_0^l \frac{1}{2}(\rho\, dy)\left(\frac{y}{l}\dot{x}\right)^2$$

$$= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}\rho\dot{x}^2 \times \frac{l}{3}$$

$$= \frac{1}{2}m\dot{x}^2 + \frac{1}{6}m_s\dot{x}^2$$

$$m_s = \rho l$$

Potential energy of the system. $= \frac{1}{2}Kx^2$

$$\text{Total} = K.E + P.E$$

$$= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m_s\frac{\dot{x}^2}{3} + \frac{1}{2}Kx^2$$

$$m\ddot{x} + \frac{m_s\ddot{x}}{3} + Kx = 0$$

$$\left(m + \frac{m_s}{3}\right)\ddot{x} + Kx = 0$$

$$\omega_n = \sqrt{\frac{K}{m + m_s/3}} \text{ rad/sec}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K}{m + m_s/3}} \text{ Hz}$$