(9) Consider again the case of system having angular motion. Let us take the example of a light, stiff rod of length l, pivoted at one end and having a concentrated mass 'm' at the other end. (simple pendulum) consider at any instant that the rod is displaced through an angle θ. The external forces acting on it for small amplitude vibration the displacement of the bob may be considered to be linear

Solve.

$$x \cong l\theta$$

∴ Now taking moment about the pivot point 'O', and applying Newton's second law,

(mass . M.I of the system about 'O') × (Angular acceleration)

= Algebraic sum of the external moments about O in the direction of angular acceleration

∴ If θ is positive anticlockwise, $\ddot{\theta}$ is also positive anticlockwise, then we have

$$J_o\ddot{\theta} = -mgx$$
$$= -mg\,l\theta$$

∴ If the bob is considered to be a concentrated mass and the rod to be of negligible mass then

$$J_o = m\,l \times l = ml^2$$

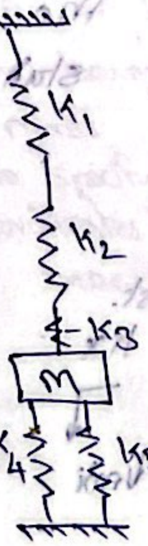
$$\therefore\ ml^2\ddot{\theta} = -mgl\theta$$

$$l\ddot{\theta} + g\theta = 0$$

$$\ddot{\theta} + \frac{g}{l}\theta = 0 \qquad \omega^2 = \frac{g}{l}$$

$$\ddot{\theta} + \omega^2\theta = 0 \qquad \omega = \sqrt{\frac{g}{l}}$$

Problem: (9)



$K_1 = 2000$ N/m

$K_2 = 1500$ N/m

$K_3 = 3000$ N/m

$K_4 = 500$ N/m

$K_5 = 500$ N/m.

To Find: mass m and the system has a natural frequency of 10 Hz = f.

Solve

If K_{e1} is the effective spring stiffness of the top three springs in series then

$$\frac{1}{K_{e1}} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}$$

$$= \frac{1}{2000} + \frac{1}{1500} + \frac{1}{3000}$$

$$\frac{1}{K_{e1}} = 0.0015$$

$$K_{e1} = 667 \text{ N/m.}$$

If K_{e2} is the effective spring stiffness of the lower two springs in parallel, then

$$K_{e2} = K_4 + K_5 = 500 + 500 = 1000$$

$$K_{e2} = 1000 \text{ N/m}$$

Now K_{e1} and K_{e2} two spring in parallel, therefore effective stiffness

$$K_e = K_{e1} + K_{e2}$$

$$= 667 + 1000 = 1667 \text{ N/m}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{K_e}{m}} = 10 \text{ Hz (given)}$$

46.2

$f_n = \frac{\omega}{2\pi}$

$\omega_n = f_n \times 2\pi$

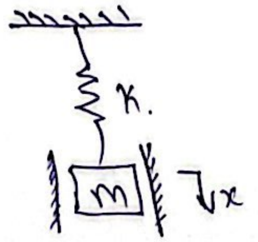
$= 10 \times 2\pi$

$= 62.8$ rad/sec

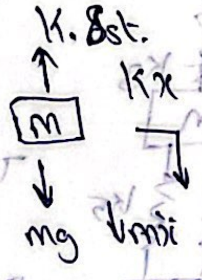
$$m = \frac{K_e}{4\pi^2 \times 10^2} = \frac{1667}{4\pi^2 \times 100}$$

$$\boxed{m = 0.422 \text{ kg}}$$

(b) Single Degree of freedom spring mass system vibrating in vertical plane, Derive and obtain the relationship between the natural frequency and static deflection.



Free body diagram



$$W = mg = K\Delta st$$

$$\Sigma F = W + K(\Delta + x)$$

$$m\ddot{x} = W - K\Delta - Kx$$

$$m\ddot{x} = -Kx$$

$$m\ddot{x} + Kx = 0$$

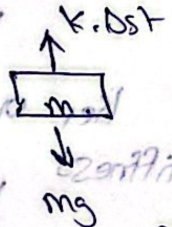
$$\ddot{x} + \left(\frac{K}{m}\right)x = 0$$

$$\ddot{x} + \omega^2 x = 0$$

$$\omega^2 = \frac{K}{m}$$

$$\omega = \sqrt{K/m} \text{ rad/sec.}$$

$$\omega = \sqrt{\frac{Kg}{mg}}$$



$$mg = K\Delta st$$

$$= \sqrt{\frac{Kg}{K \cdot \Delta st}}$$

$$= \sqrt{\frac{g}{\Delta st}}$$

$$= \sqrt{\frac{9.81}{\Delta st}}$$

$$\omega = \frac{3.132}{\sqrt{\Delta st}}$$

Result:

i. $m\ddot{x} + Kx = 0$

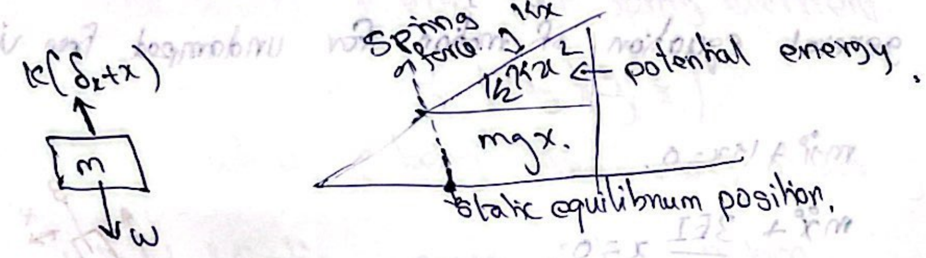
ii. $\omega = \sqrt{K/m}$

iii. $\omega = \frac{3.132}{\sqrt{\Delta st}}$

some systems especially those involving continuous elastic members; have an infinite number of degrees of freedom.

Ex: Cantilever beam since the beam has an infinite number of mass points, we need an infinite number of coordinates to specify its deflected configuration. The infinite number of coordinates defines its elastic deflection curve. Thus the cantilever beam has an infinite number of degrees of freedom.

Spring:-



Since the spring force is mg at $x=0$, the potential energy of the spring under the deformation x will be $mgx+\frac{1}{2}kx^2$. The potential energy of the system due to change in elevation of the mass is $-mgx$

$\therefore$ U = potential energy of the spring + change in potential energy due to change in elevation of the mass m

$$= mgx + \frac{1}{2}kx^2 - mgx$$

$$= \frac{1}{2}kx^2$$