Energy method :-

$$P.E = mgh$$

$$= mgl\theta$$

$$K.E = \frac{1}{2}mv^2$$

$$= \frac{1}{2}J\dot{\theta}^2$$

$\therefore$ P.E + K.E = Total energy

$$= mgl\theta + \frac{1}{2}J\dot{\theta}^2$$

$$\frac{d}{dt}(T) = 0$$

$$\frac{d(T)}{dt} = mgl\theta\dot{\theta} + \frac{1}{\cancel{2}}J\,\cancel{2}\,\dot{\theta}\ddot{\theta}$$

$$0 = mgl\theta\dot{\theta} + J\dot{\theta}\ddot{\theta}$$

$$0 = \dot{\theta}(mgl\theta + J\ddot{\theta})$$

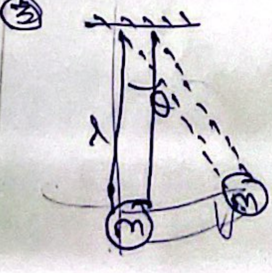
$$0 = \dot{\theta}(mgl\theta + ml^2\ddot{\theta})$$

$$0 = \cancel{m}gl\theta + \cancel{m}l^{\cancel{2}}\ddot{\theta}$$

$$\ddot{\theta}l + g\theta = 0$$

$$\boxed{\ddot{\theta} + \left(\frac{g}{l}\right)\theta = 0}$$

③



$$J\ddot{\theta} = -m_b\,gl\sin\theta$$

$$J\ddot{\theta} + m_b\,gl\theta = 0$$

$$\left(m_b l^2 + m_r\frac{l^2}{3}\right)\ddot{\theta} + m_b\,gl\theta = 0.$$

$$\ddot{\theta} + \frac{m_b g l}{\left(m_b l^2 + m_r \frac{l^2}{3}\right)} \theta = 0.$$

Energy method

$$P.E = mgh = mgl\theta$$

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}J\dot{\theta}^2$$

$$\frac{d(T)}{dt} = mgl\theta + \frac{1}{2}J\dot{\theta}^2$$

$$= mgl\theta\dot{\theta} + \frac{1}{2}J\dot{\theta}^2$$

$$= mgl\theta\dot{\theta} + \frac{1}{2}J2\dot{\theta}\ddot{\theta}$$

$$= mgl\theta\dot{\theta} + J\dot{\theta}\ddot{\theta}$$

$$0 = \dot{\theta}(mgl\theta + J\ddot{\theta})$$

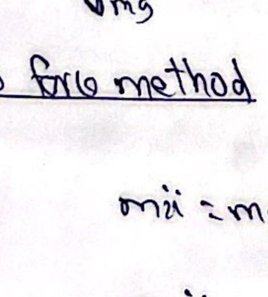
$$(mgl\theta + J\ddot{\theta}) = 0$$

$$\ddot{\theta} + \frac{mgl}{J}\theta = 0.$$

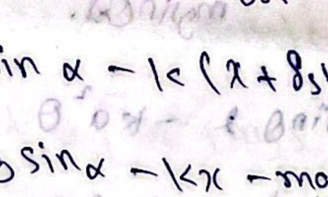
$$J = \left(m_b l^2 + m_r \frac{l^2}{3}\right)$$

$$\therefore \ddot{\theta} + \frac{mgl}{\left(m_b l^2 + m_r \frac{l^2}{3}\right)}\theta = 0.$$

(4)



Newton's force method



$$m\ddot{x} = mg\sin\alpha - kx - k\delta_{st}$$

$$m\ddot{x} = mg\sin\alpha - k(x + \delta_{st}) \qquad k\delta_{st} = mg\sin\alpha$$

$$= mg\sin\alpha - kx - mg\sin\alpha$$

$$m\ddot{x} = -kx$$

$$\boxed{m\ddot{x} + kx = 0.}$$

D'Alembert's method:

The system is statically unstable due to acceleration $\ddot{x}$. It can be made stable by applying an opposite inertia force $m\ddot{x}$

$$\therefore m\ddot{x}+kx=0.$$

Energy method:

$$P.E=\frac{1}{2}kx^2$$

$$K.E=\frac{1}{2}mv^2.$$

$$=\frac{1}{2}m\dot{x}^2$$

Total Energy.

$$=\frac{1}{2}kx^2+\frac{1}{2}m\dot{x}^2$$

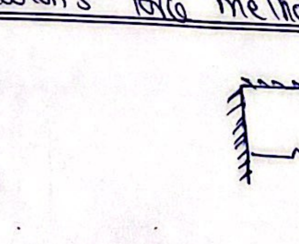
$$\frac{d(T)}{dt}=\dot{x}(kx+m\ddot{x})=0$$

$$\boxed{kx+m\ddot{x}=0}$$

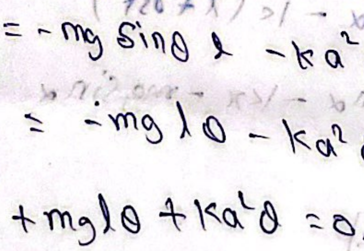
$$\ddot{x}+\omega^2x=0$$

$$\therefore \omega^2=\frac{k}{m}$$

(5)



Newton's force method:



$mgl\sin\theta$.

$$J\ddot{\theta}=-mg\sin\theta\, l-ka^2\theta$$

$$J\ddot{\theta}=-mgl\theta-ka^2\theta \qquad \sin\theta=\theta.$$

$$J\ddot{\theta}+mgl\theta+ka^2\theta=0.$$

$$ml^2\ddot{\theta} + \theta(mgl + ka^2) = 0$$

$$\ddot{\theta} + \left[\frac{mgl + ka^2}{ml^2}\right]\theta = 0$$

$ka\theta$ = Force

Σ xtension = $a\theta$

moment = $ka^2\theta$.

D'Alembert's method:-

$$J\ddot{\theta} + mgl\theta + ka^2\theta = 0.$$

$$\ddot{\theta} + \left[\frac{mgl + ka^2}{J}\right]\theta = 0$$

$$\ddot{\theta} + \left[\frac{mgl + ka^2}{ml^2}\right]\theta = 0$$

Energy method:-

$$P.E = \frac{1}{2}kx^2 + mgh$$

$$= \frac{1}{2}k(a\theta)^2 + mgl\theta$$

$$K.E = \frac{1}{2}mv^2$$

$$= \frac{1}{2}J\dot{\theta}^2$$

$$\text{Total Energy} = \frac{1}{2}k(a\theta)^2 + mgl\theta + \frac{1}{2}J\dot{\theta}^2$$

$$\frac{dT}{dt} = \frac{d}{dt}\left(\frac{1}{2}ka^2\theta^2 + mgl\theta + \frac{1}{2}J\dot{\theta}^2\right) = 0$$

$$= \frac{1}{2}ka^2 2\theta\dot{\theta} + mgl\theta\dot{\theta} + \frac{1}{2}J2\dot{\theta}\ddot{\theta}$$

$$\Rightarrow ka^2\theta\dot{\theta} + mgl\theta\dot{\theta} + J\dot{\theta}\ddot{\theta} = 0$$

$$\dot{\theta}\left[ka^2\theta + mgl\theta + J\ddot{\theta}\right] = 0$$

$$\left[ka^2 + mgl\right]\theta + J\ddot{\theta} = 0$$

$$J\ddot{\theta} + \theta\left[ka^2 + mgl\right] = 0$$

$$\ddot{\theta} + \left[\frac{ka^2 + mgl}{J}\right]\theta = 0.$$

$J = ml^2$