Solution:

$$x_1 = 3(\sin\omega t \cos 30^\circ + \cos\omega t \sin 30^\circ)$$

$$x_2 = 2(\cos\omega t \cos 15^\circ + \sin\omega t \sin 15^\circ)$$

Adding.

$$x_1 + x_2 = 2.598 \sin\omega t + 1.5 \cos\omega t + 1.93 \cos\omega t + 0.5176 \sin\omega t$$

$$x_1 + x_2 = 3.11 \sin\omega t + 3.43 \cos\omega t \longrightarrow (1)$$

$$x = A\sin(\omega t + \phi)$$

$$= A(\sin\omega t \cos\phi + \cos\omega t \sin\phi) \longrightarrow (2)$$

(1) = (2) Comparing

$$A\cos\phi = 3.11$$

$$A\sin\phi = 3.43$$

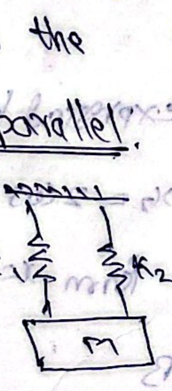
$$\tan\phi = \frac{3.43}{3.11} \Rightarrow \phi = 47.8^\circ$$

$$A = \sqrt{A_1^2 + A_2^2} = 4.63$$

EQUIVALENT STIFFNESS OF SPRING COMBINATION.

Certain system have more than one spring. The springs are joined in series or parallel or both. They can be replaced by a single spring of the same stiffness as they allow the same stiffner jointly.

Springs in parallel.



The deflection of individual spring is equal to the deflection of the system

$$\therefore K_1 x + K_2 x = K_e x$$

$$K_e = K_1 + K_2$$

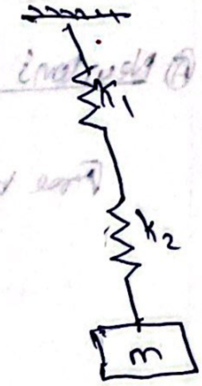
K_e = equivalent stiffness of the system

x = deflection

K_1, K_2 = stiffness, $K_e x$ = force on the system.

Springs in series :-

The total deflection of the system is equal to the sum of deflection of individual springs.



$$x = x_1 + x_2 + x_3 \cdots\cdots$$

$$\frac{\text{Force}}{k_e} = \frac{\text{Force}}{k_1} + \frac{\text{Force}}{k_2} + \cdots\cdots$$

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \cdots$$

Problems : 3 to 7

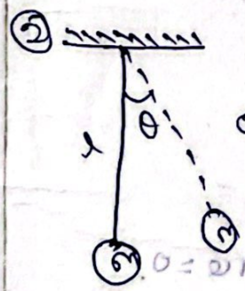
Find the Differential equation for the given systems.

① A mass 'm' is suspended by an spring of stiffness 'k' if the mass is pulled down by a distance of 'x'. Find the Governing diff eqn.

displacement only
No force.

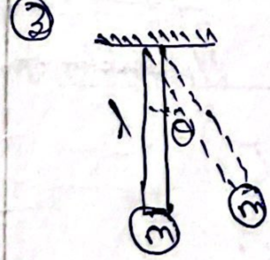
by (i) Newton's force method
(ii) D'Alembert's method
(iii) Energy method.

②

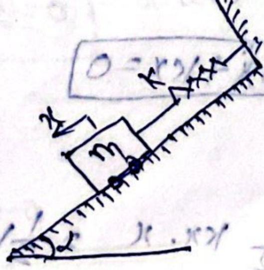


A mass 'm' is suspended from a point and it oscillates by an angle 'θ'. The length of the string is 'l'. Find the Governing equation.

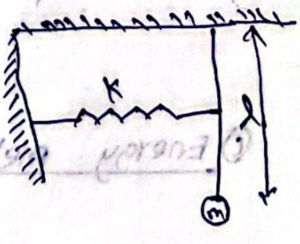
③



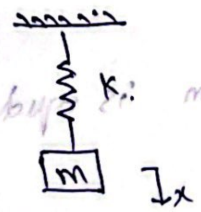
④



⑤

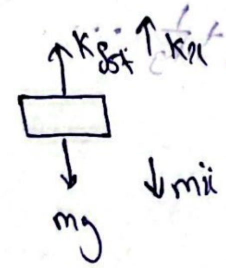


①.



Ⓐ Newton's force method

Free body dia



$$k\delta_{st} = -mg$$

$$m\ddot{x} = mg - k\delta_{st} - kx$$

$$m\ddot{x} = -kx$$

$$\boxed{m\ddot{x} + kx = 0}$$

Ⓑ D'Alembert's method

Inertia force $= m\ddot{x}$

Spring force $= kx$

In force + Sp force = 0.

$$\therefore \boxed{m\ddot{x} + kx = 0}$$

Ⓒ Energy method:

$$P.E = \frac{1}{2} kx . x = \frac{1}{2}kx^2$$

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m\dot{x}^2$$

Total Energy (T) = P.E + K.E.

$$= \frac{1}{2}kx^2 + \frac{1}{2}m\dot{x}^2$$

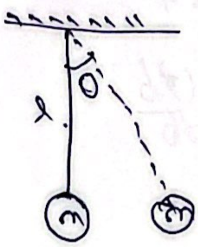
$$\frac{d}{dt}(T) = 0.$$

$$\frac{d(T)}{dt} = \frac{1}{2}k.\,2x\dot{x} + \frac{1}{2}m\,2\dot{x}\ddot{x}$$

$$= kx\dot{x} + m\dot{x}\ddot{x}$$

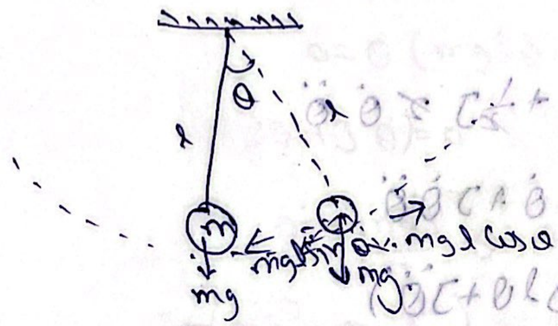
$\dot{x}(kx + m\ddot{x}) = 0$ As $\dot{x} \neq 0$

$$\boxed{m\ddot{x} + kx = 0}$$

② 

(mass of the rod negligible.

<u>Newton's force method.</u>



$J\ddot{\theta}$ = Restoring Torque force.

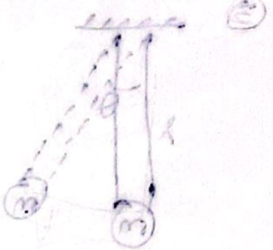
$\therefore J\ddot{\theta} = -mgl\sin\theta$ $\qquad \sin\theta \approx \theta$

$J\ddot{\theta} = -mgl\theta$ $\qquad J = ml^2$

$ml^2\ddot{\theta} + mgl\theta = 0$

$\boxed{\ddot{\theta} + \left(\frac{g}{l}\right)\theta = 0}$ G.D.E.

<u>D'Alembert's method:</u>



$J\ddot{\theta} + mgl\theta = 0$

$\ddot{\theta} + \frac{mgl}{ml^2}\theta = 0$

$\ddot{\theta} + \left(\frac{g}{l}\right)\theta = 0$

$\ddot{\theta} + \omega\theta = 0$