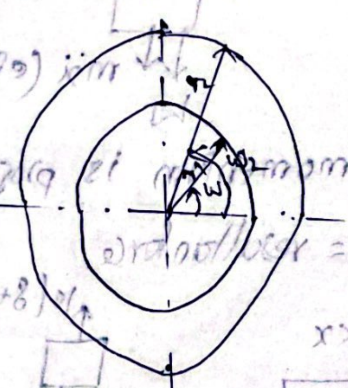
BEATS PHENOMENON:-

When two harmonic motions pass through some point in a medium simultaneously, the resultant displacement at that point is the vector sum of the displacement due to two component motions. This superposition of motion is called interference.

The phenomenon of beat occurs as a result of interference between two waves of slightly different frequencies moving along the same straight line in the same direction. [Beats phenomenon occur when they are two rotating vector of different amplitude and slightly varying phase angle.

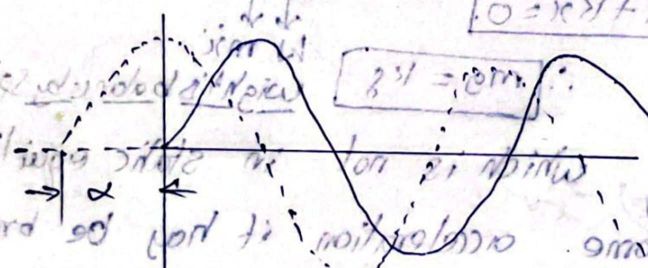


$$v_1 = v_2$$

$$\omega_1 t \neq \omega_2 t$$

$$\omega_2 t = \omega_1 t + \Delta\omega t$$

$$\omega_2 - \omega_1 = \Delta\omega$$



Consider that at particular time the two wave motion are in same phase, At this stage the resultant amplitude of vibration will be maximum. On the other hand, when the two motions are not in phase with each other, they produce minimum amplitude of vibration.

Again after some time the two motion are in phase and produce maximum amplitude and minimum amplitude. This keep on changing from maximum to minimum. This phenomenon is known as beat.

Let us consider two waves of the same amplitude A and slightly different frequencies ω_1 and ω_2. If x_1 and x_2 are the displacement of the two waves and phase angle at any time (t) then

$$x_1 = A \sin \omega_1 t$$

$$x_2 = A \sin \omega_2 t$$

Resultant $x_1 + x_2$: $x = A(\sin \omega_1 t + \sin \omega_2 t)$

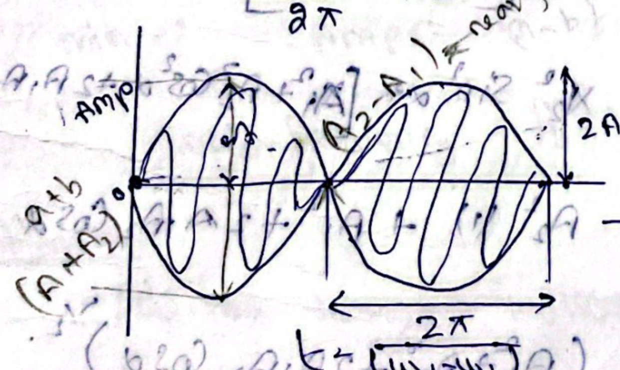
$$x = 2A \sin \frac{(\omega_1+\omega_2)t}{2} \cos \frac{(\omega_1-\omega_2)t}{2}$$

$$x = 2A \cos \frac{(\omega_1-\omega_2)t}{2} \sin \frac{(\omega_1+\omega_2)t}{2}$$

$$\therefore \quad x = B \sin \frac{(\omega_1+\omega_2)t}{2} \longrightarrow ①$$

$$\text{where } B = 2A \cos \frac{(\omega_1-\omega_2)t}{2} \longrightarrow ②$$

eqn ① represent a simple harmonic motion whose amplitude is B. is max $2A$ and min is zero. The frequency of beat is $\frac{\omega_1+\omega_2}{2\pi}$ Hz.



$$t = \frac{2\pi}{\omega}$$

$$t = \frac{2\pi}{(\omega_1+\omega_2)}$$

$$f = \frac{1}{2\pi}\omega_n$$

$$f = \frac{\omega_1-\omega_2}{2\pi}$$

To find the phase angle and Amplitude.

$$x_1 = A_1 \sin \omega t \longrightarrow ①$$

$$x_2 = A_2 \sin(\omega t + \alpha) \longrightarrow ②$$

$$x_1 + x_2 = A \sin(\omega t + \beta) \longrightarrow ③$$

② $\Rightarrow$ $x_2 = A_2 \left[\sin\omega t \cos\alpha + \sin\alpha \cos\omega t \right]$

$x_1 = A_1 \sin\omega t$.

$$x_1 + x_2 = A\left[\sin\omega t \cos\beta + \sin\beta \cos\omega t\right]$$

$$A\left[\sin\omega t \cos\beta t + \cos\omega t \sin\beta\right] = x_2 + x_1$$

$$= (A_1 + A_2 \cos\alpha)\sin\omega t + A_2 \sin\alpha \cos\omega t$$

$$A \cancel{\sin\omega t} \cos\beta = (A_1 + A_2\cos\alpha)\cancel{\sin\omega t}$$

$$A \cos\cancel{\omega t} \sin\beta = A_2 \sin\alpha \cancel{\cos\omega t}$$

$$A\cos\beta = (A_1 + A_2\cos\alpha)$$

① $\leftarrow$ $A\sin\beta = A_2 \sin\alpha$.

② $\leftarrow$ $$\tan\beta = \frac{A_2 \sin\alpha}{(A_1 + A_2)\cos\alpha}.$$

$$\beta = \tan^{-1}\left[\frac{A_2 \sin\alpha}{(A_1 + A_2)\cos\alpha}\right]$$

$$\therefore A^2\left[\sin^2\beta + \cos^2\beta\right] = A_2^2 \sin^2\alpha + \left[A_1^2 + A_2^2\cos^2\alpha + 2A_1A_2\cos\alpha\right]$$

$$A^2 = A_2^2(1) + A_1^2 + 2A_1A_2\cos\alpha$$

amplitude. $$A = (A_1^2 + A_2^2 + 2A_1A_2\cos\alpha)^{1/2}.$$

Case(i) When the two sinusoidal motion are in phase, then phase difference $\alpha = 0^\circ$

Then $$A = \left[A_1^2 + A_2^2 + 2A_1A_2\cos(0)\right]^{1/2}$$

$$= \sqrt{A_1^2 + A_2^2 + 2A_1A_2}$$

$$= \sqrt{(A_1 + A_2)^2} = (A_1 + A_2).$$

Case II: when two sinusoidal motions are out of phase then phase difference (i.e. $\alpha = 180°$)

$$\text{Resultant amplitude} = \sqrt{A_1^2+A_2^2+2A_1A_2\cos\alpha}$$

$$= \sqrt{A_1^2+A_2^2+2A_1A_2\cos(180)}$$

$$= \sqrt{A_1^2+A_2^2-2A_1A_2}$$

$$= \sqrt{(A_1-A_2)^2}$$

$$= (A_1-A_2)$$

∴ Amp changing from max $(A+B)$ to min $(A-B)$ with a frequency equal to the difference between the individual component frequencies.

Problem ①:

A body describes simultaneously two motions, $x_1 = 3\sin 40t$, $x_2 = 4\sin 41t$. What is the maximum and minimum amplitude of combined motion and what is the beat frequency.

Solution:
If a body is subjected to two harmonic motion

$x_1 = a\sin\omega_1 t$

$x_2 = b\sin\omega_2 t$

maximum Amp $= (a+b) = 3+4 = 7$

min Amp $= (a-b) = 4-3 = 1$

Beat frequency $f_b = \frac{\omega}{2\pi} = \frac{\omega_1-\omega_2}{2\pi} = \frac{41-40}{2\pi} = \frac{1}{2\pi}$ Hz.

Problem ②.

Add two harmonic motions expressed by

$x_1 = 3\sin(\omega t+30°)$; $x_2 = 2\cos(\omega t-15°)$

express the result in the form $x = A\sin(\omega t+\phi)$.

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$.