PERIODIC AND HARMONIC MOTION:-

The motion which repeats itself after an equal interval of time is known as periodic motion. The equal interval is called time period. If we consider a motion of the type $x_1 = A_1 \sin\omega t$, here ω is the natural frequency and the motion will be repeated after $2\pi/\omega$ time. The harmonic motion is one of the form of periodic motion. The harmonic motion is represented in terms of circular sine and cosine functions.

All harmonic motions are periodic in nature but vice-versa is not always true. In the equation

$$x_1 = A_1 \sin\omega t \longrightarrow (1)$$

where x_1 = is the displacement

A_1 = the amplitude

The velocity $\Rightarrow \dot{x}_1 = \frac{dx_1}{dt} = A_1 \omega \cos\omega t \longrightarrow (2)$

The acceleration $\Rightarrow \ddot{x}_1 = -\omega^2 x_1 \longrightarrow (3)$

$\therefore$ from equ (3) The acceleration in a simple harmonic motion is always proportional to its displacement and directed towards a particular fixed point.

It is shown that when harmonic motions of same period are added the resultant harmonic motion of same period is obtained.

ADDITION OF HARMONIC MOTION:-

When we add two harmonic motions of the same frequency we get the resultant motion as harmonic

$\therefore$ Let us have two harmonic motions of Amplitudes A_1 and A_2, the same frequency ω and phase different ϕ as.

$$x_1 = A_1 \sin\omega t \longrightarrow (1)$$

$$x_2 = A_2 \sin(\omega t + \phi) \longrightarrow (2)$$

The resultant motion is given by adding the above equ.

$$x = x_1 + x_2$$

$$= A_1 \sin\omega t + A_2 \sin(\omega t + \phi) \longrightarrow (3)$$

$$= A_1 \sin\omega t + A_2 (\sin\omega t \cos\phi + \sin\phi \cos\omega t)$$

$$= \sin\omega t \, [A_1 + A_2 \cos\phi] + A_2 \cos\omega t \sin\phi$$

Assuming

$$\left.\begin{aligned} A_1 + A_2 \cos\phi &= A \cos\theta \\ A_2 \sin\phi &= A \sin\theta \end{aligned}\right\} \longrightarrow (4)$$

$$\therefore x = A \sin\omega t \cos\theta + A \sin\theta \cos\omega t$$

$$= A \sin(\omega t + \theta) \longrightarrow (5)$$

$\therefore$ The above (5) shows that the resultant displacement is also simple harmonic motion (SHM) of amplitude A and phase θ. <u>To find 'A'</u>:

$$4 \Rightarrow \quad A^2 (\cos^2\theta + \sin^2\theta) = (A_1 + A_2 \cos\phi)^2 + A_2^2 \sin^2\phi.$$

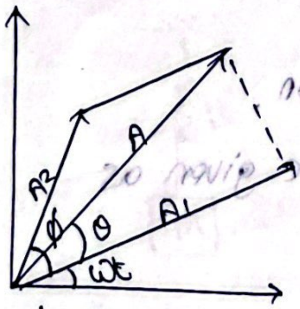
$$= A_1^2 + A_2^2 \cos^2\phi + 2A_1A_2\cos\phi + A_2^2 \sin^2\phi$$

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi.$$

$$A = \sqrt{A_1^2 + A_2^2 + 2AA_2\cos\phi}.$$

To find phase differen, θ.

$$\boxed{\tan\theta = \frac{A_2 \sin\phi}{A_1 + A_2\cos\phi}}$$



$$\theta = \tan^{-1}\left[\frac{A_2 \sin\phi}{A_1 + A_2 \cos\phi}\right].$$

$\hookrightarrow$ Graphical method for the addition of two SHM.